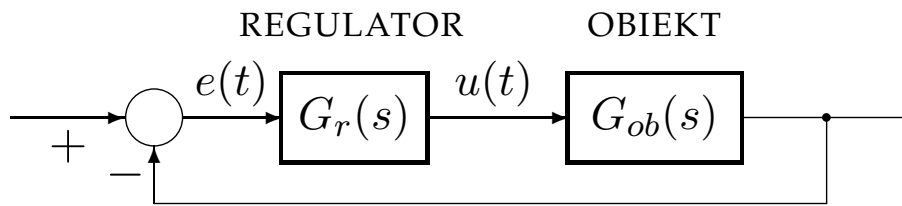


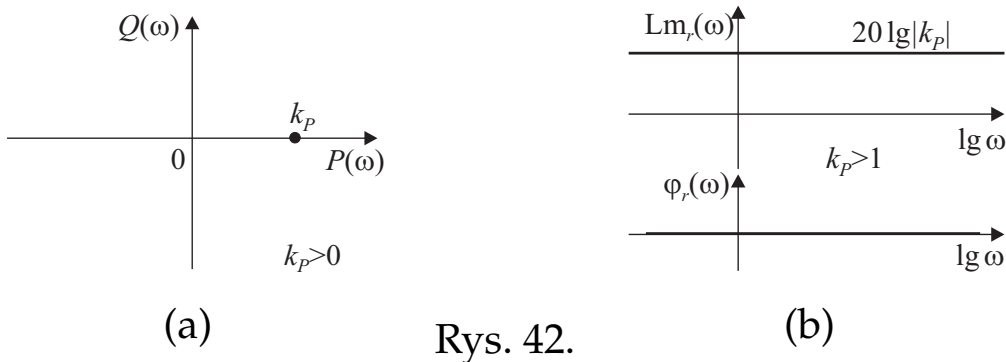
14. Regulatory liniowe o wyjściu ciągłym



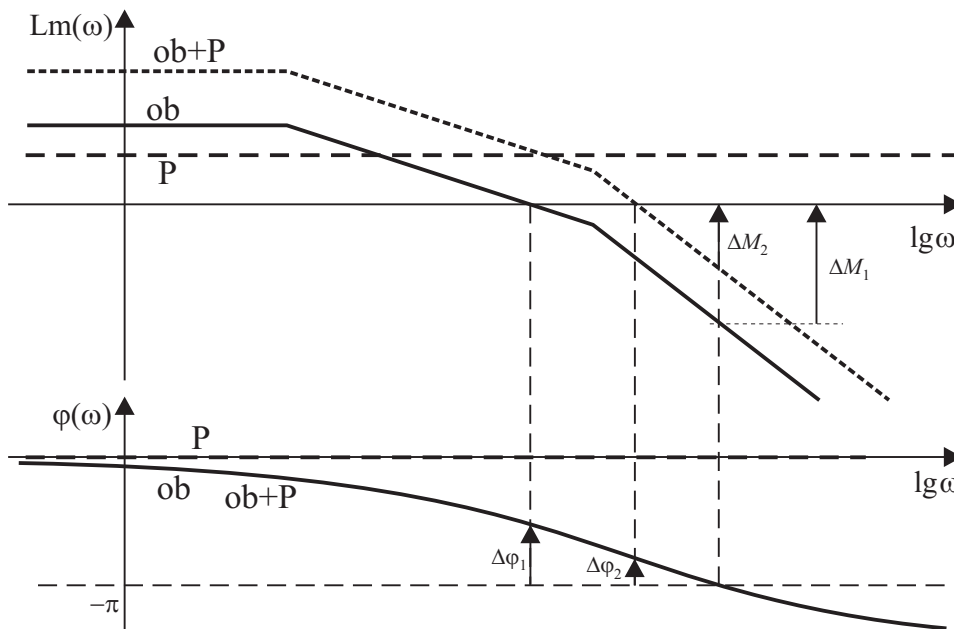
Rys. 41.

a) regulator typu P (proporcjonalny):

$$G_r(s) = \frac{U(s)}{E(s)} = k_p, \quad G_r(j\omega) = G_r(s)|_{s=j\omega} = k_p + j0, \quad k_p > 0$$

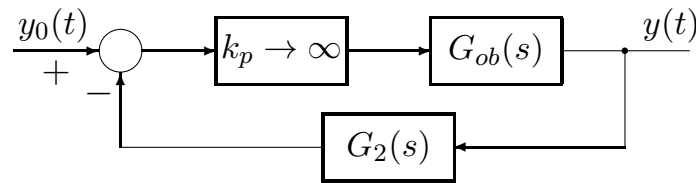


Rys. 42.



Rys. 43.

$$e_u = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \frac{1}{1 + k_p k_{ob}}$$



Rys. 44.

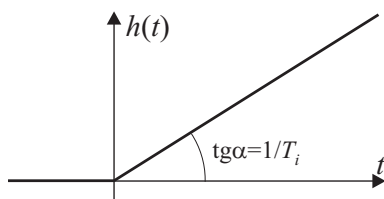
$$G(s) = \frac{Y(s)}{Y_0(s)} = \frac{k_p G_{ob}(s)}{1 + k_p G_{ob}(s) G_2(s)}$$

$$\lim_{k_p \rightarrow \infty} G(s) = \lim_{k_p \rightarrow \infty} \frac{G_{ob}(s)}{1/k_p + G_{ob}(s) G_2(s)} = \frac{1}{G_2(s)} \quad (45)$$

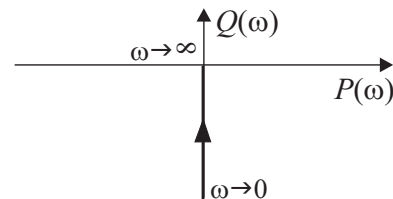
b) regulator typu I (całkujący):

$$G_r(s) = \frac{U(s)}{E(s)} = \frac{1}{T_i s}, \quad G_r(j\omega) = G_r(s)|_{s=j\omega} = \frac{1}{jT_i \omega} = 0 - j \frac{1}{T_i \omega}$$

$$Lm_r(\omega) = -20 \log(T_i) - 20 \log(\omega), \quad \varphi_r(\omega) = -\pi/2$$

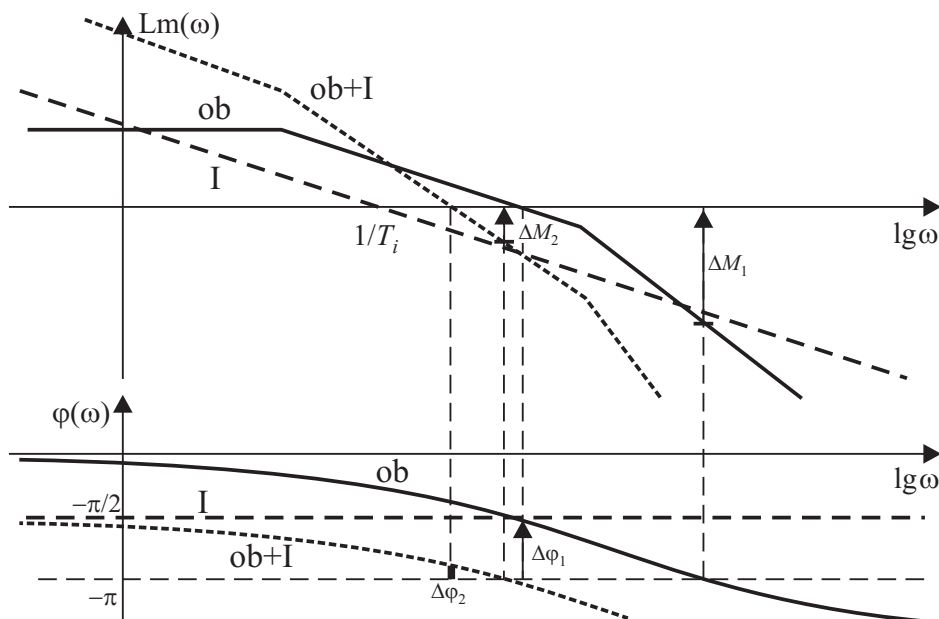


(a)



(b)

Rys. 45.

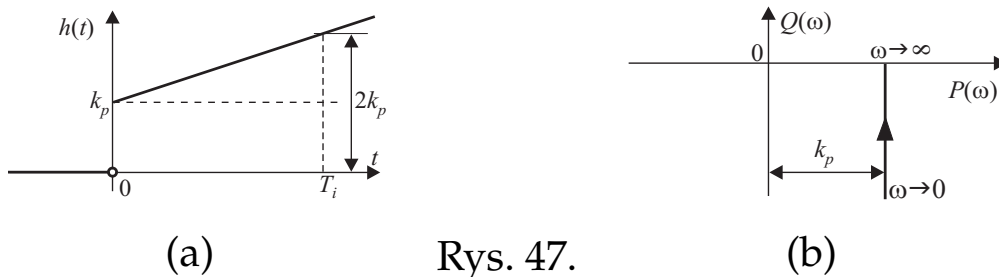


Rys. 46.

c) regulator typu PI (proporcjonalno-całkujący)

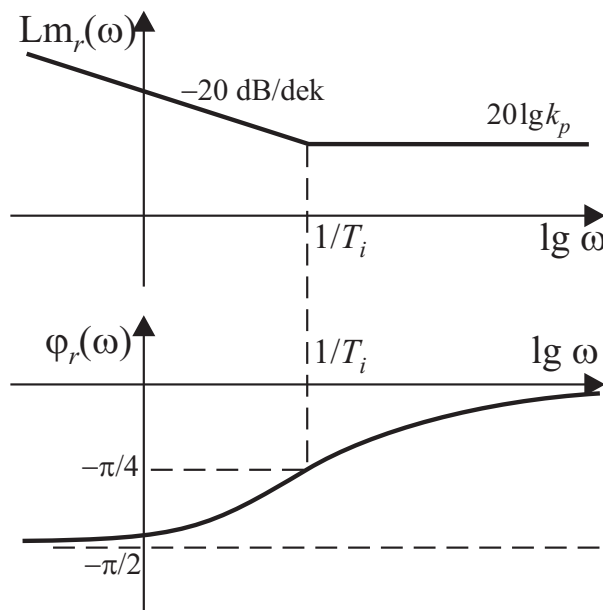
$$G_r(s) = \frac{U(s)}{E(s)} = k_p \left(1 + \frac{1}{T_i s} \right) \Rightarrow u(t) = k_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau \right) \quad (46)$$

$$h(t) = \mathcal{L}^{-1} \left\{ G_r(s) \frac{1}{s} \right\} = k_p \left(1 + \frac{t}{T_i} \right) \mathbb{1}(t) \quad (47)$$

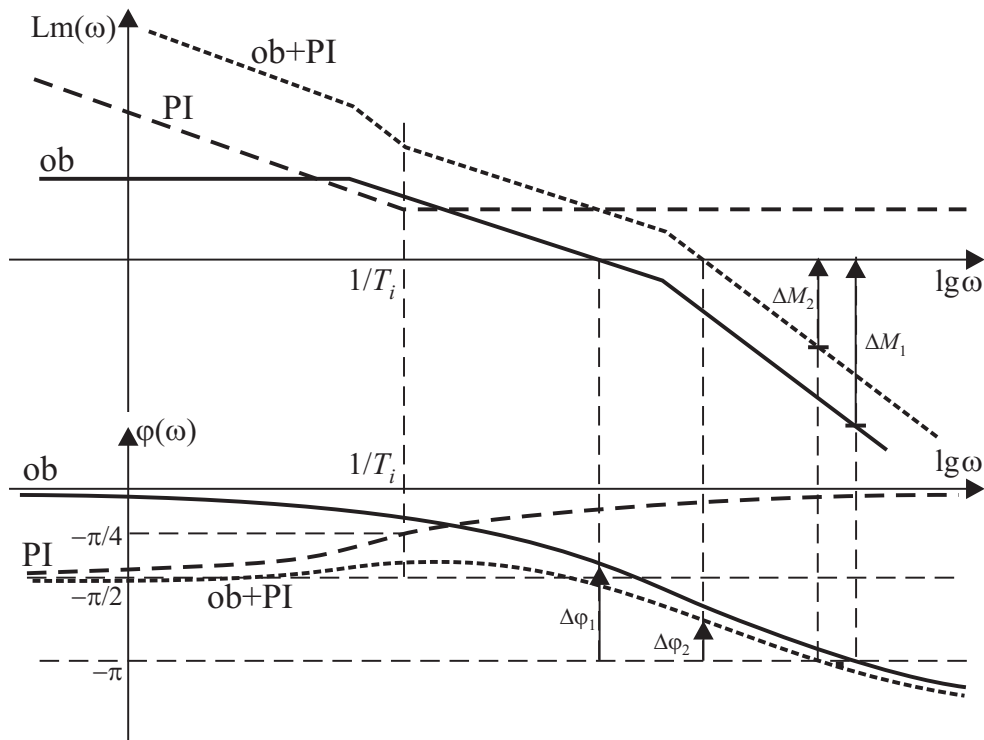


$$G_r(j\omega) = k_p \left(1 + \frac{1}{jT_i\omega} \right) = k_p \left(1 - j \frac{1}{T_i\omega} \right) \Rightarrow \varphi_r(\omega) = -\arctg \frac{1}{T_i\omega}$$

$$Lm_r(j\omega) = 20 \lg \left(k_p \sqrt{1 + \frac{1}{T_i^2 \omega^2}} \right) \approx \begin{cases} 20 \lg k_p - 20 \lg(T_i\omega), & \omega < 1/T_i \\ 20 \lg k_p, & \omega \geq 1/T_i \end{cases}$$



Rys. 48.



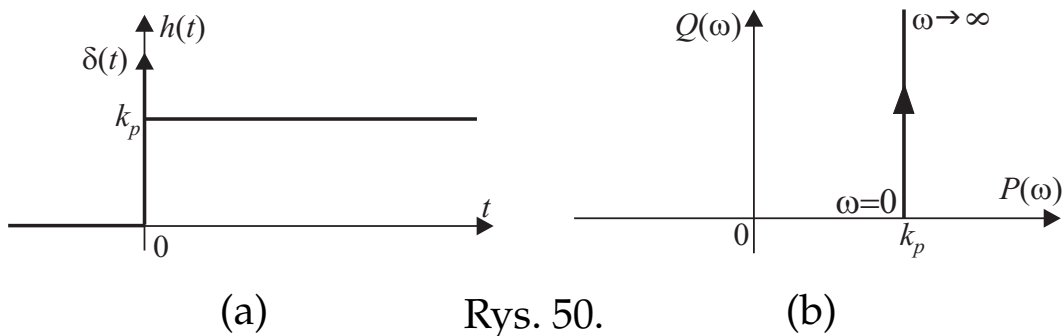
Rys. 49.

d) regulator typu PD (proporcjonalno-różniczkujący)

* regulator typu PD idealny

$$G_r(s) = \frac{U(s)}{E(s)} = k_p(1 + T_d s) \Rightarrow u(t) = k_p \left(e(t) + T_d \frac{de(t)}{dt} \right) \tag{48}$$

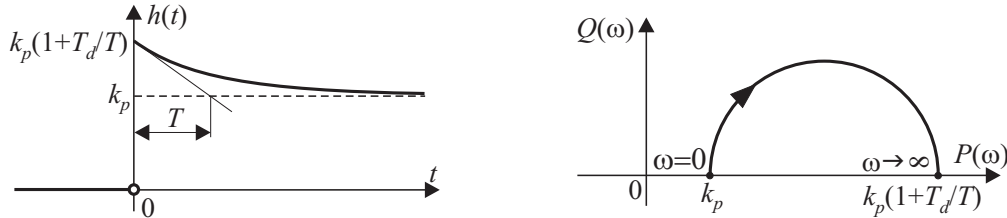
$$h(t) = k_p(\mathbb{1}(t) + T_d \delta(t)), \quad G_r(j\omega) = G_r(s)|_{s=j\omega} = k_p(1 + jT_d \omega)$$



Rys. 50.

* regulator typu PD rzeczywisty

$$G_r(s) = \frac{U(s)}{E(s)} = k_p \left(1 + \frac{T_d s}{1 + T s} \right) \Rightarrow h(t) = k_p \left(1 + \frac{T_d}{T} e^{-\frac{t}{T}} \right) \mathbb{1}(t) \quad (49)$$



(a) Rys. 51. (b)

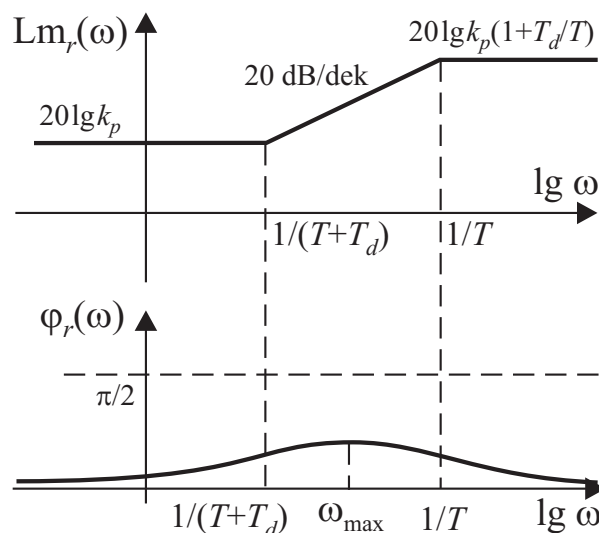
$$G_r(j\omega) = k_p \left(1 + \frac{jT_d\omega}{1 + jT\omega} \right) = k_p \frac{1 + j(T + T_d)\omega}{1 + jT\omega}$$

$$Lm_r(\omega) = 20 \lg k_p + 20 \lg \sqrt{1 + (T + T_d)^2 \omega^2} - 20 \lg \sqrt{1 + T^2 \omega^2} \approx$$

$$\approx \begin{cases} 20 \lg k_p, & \omega < 1/(T + T_d) \\ 20 \lg k_p + 20 \lg((T + T_d)\omega), & 1/(T + T_d) \leq \omega < 1/T \\ 20 \lg k_p + 20 \lg((T + T_d)\omega) - 20 \lg(T\omega), & \omega \geq 1/T \end{cases}$$

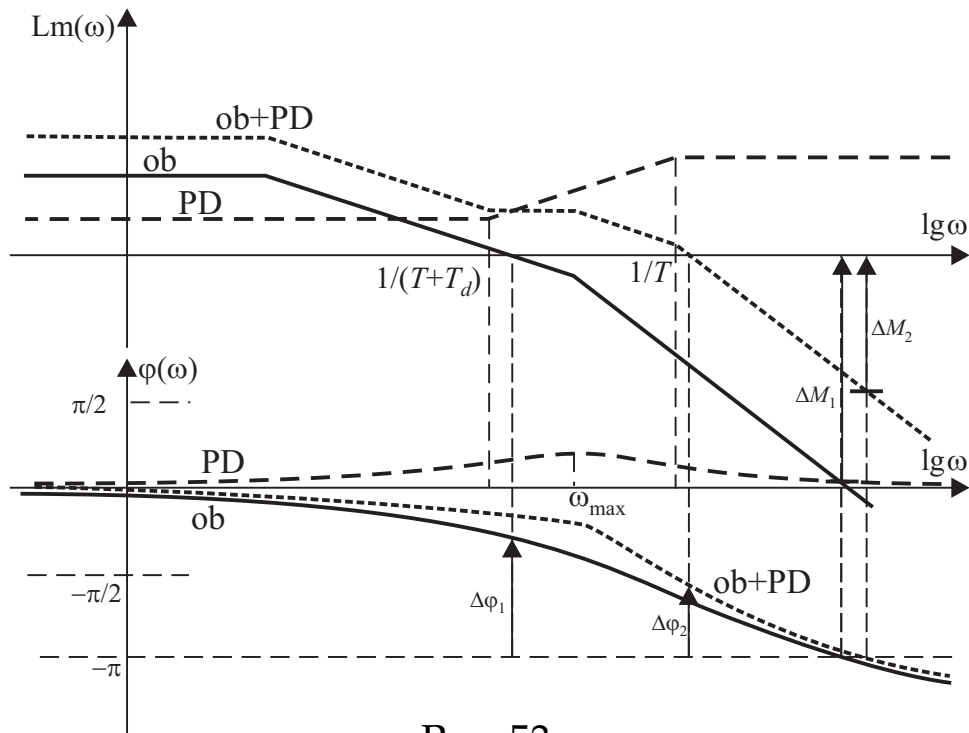
$$= 20 \lg(k_p(T+T_d)/T) = 20 \lg(k_p(1+T_d/T))$$

$$\varphi_r(\omega) = \text{arctg}((T + T_d)\omega) - \text{arctg}(T\omega), \quad \omega_{\max} = \frac{1}{\sqrt{T(T + T_d)}}$$



Rys. 52.

$$\frac{1}{sT_i}(1 + T_d s) = \frac{T_d}{T_i} \left(1 + \frac{1}{T_d s} \right)$$



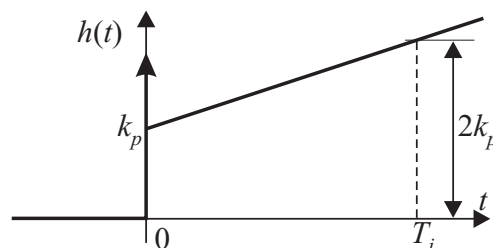
Rys. 53.

e) regulator typu PID (proporcjonalno-całkująco-różniczkujący)
 * regulator typu PID idealny

$$G_r(s) = \frac{U(s)}{E(s)} = k_p \left(1 + \frac{1}{T_i s} + T_d s \right) \tag{50}$$

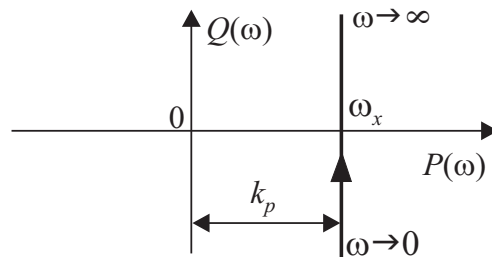
$$u(t) = k_p \left(e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$

$$h(t) = \mathcal{L}^{-1} \left\{ G_r(s) \frac{1}{s} \right\} = k_p \left(1 + \frac{t}{T_i} + T_d \delta(t) \right) \mathbb{1}(t) \tag{51}$$



Rys. 54.

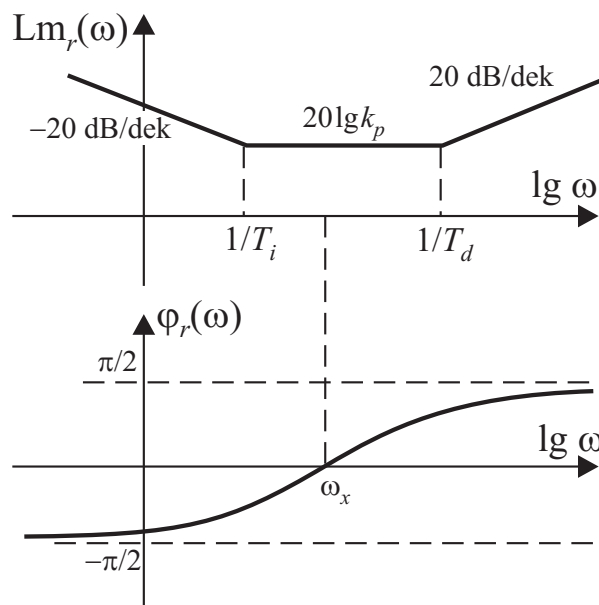
$$G_r(j\omega) = k_p \left(1 + \frac{1}{jT_i\omega} + jT_d\omega \right) = k_p \left[1 + j \left(T_d\omega - \frac{1}{T_i\omega} \right) \right]$$



Rys. 55. $\omega_x = 1/\sqrt{T_i T_d}$

$$\begin{aligned} \text{Lm}_r(\omega) &= 20 \lg k_p + 20 \lg \sqrt{1 + \left(T_d\omega - \frac{1}{T_i\omega} \right)^2} = \{ \text{zał. } T_i > T_d \} \approx \\ &\approx \begin{cases} 20 \lg k_p - 20 \lg(T_i\omega), & \omega < 1/T_i \\ 20 \lg k_p, & 1/T_i \leq \omega < 1/T_d \\ 20 \lg k_p + 20 \lg(T_d\omega), & \omega \geq 1/T_d \end{cases} \end{aligned}$$

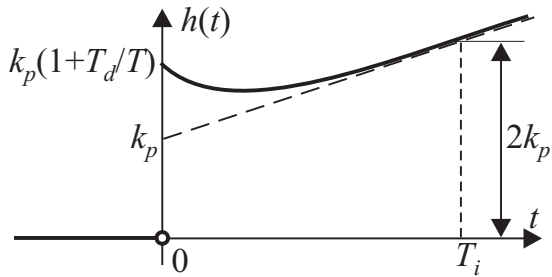
$$\varphi_r(\omega) = \text{arctg} \left(T_d\omega - \frac{1}{T_i\omega} \right)$$



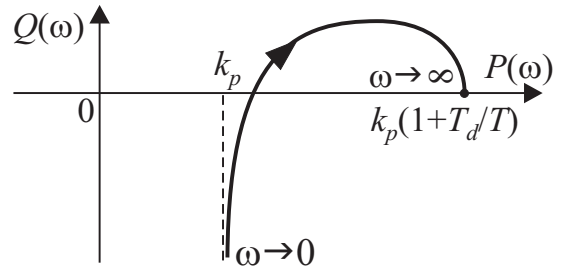
Rys. 56. zał. $T_i > T_d$

* regulator typu PID rzeczywisty

$$G_r(s) = k_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T s} \right) \Rightarrow h(t) = k_p \left(1 + \frac{t}{T_i} + \frac{T_d}{T} e^{-t/T} \right) \mathbb{1}(t) \quad (52)$$

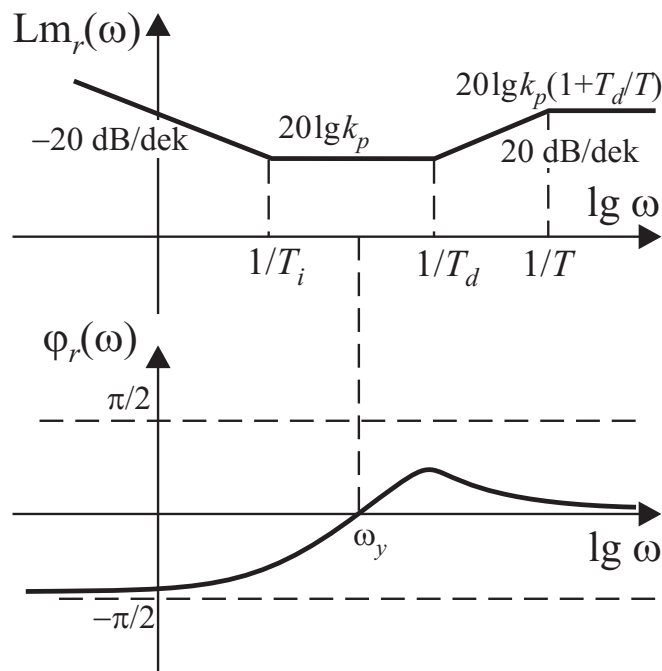


(a)

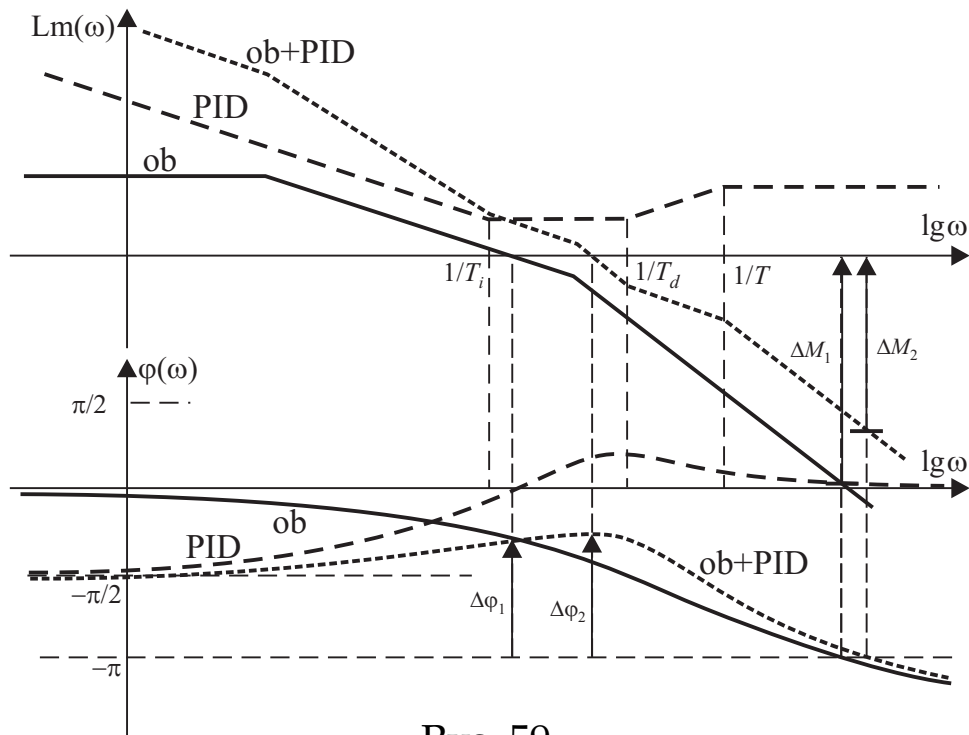


(b)

Rys. 57.



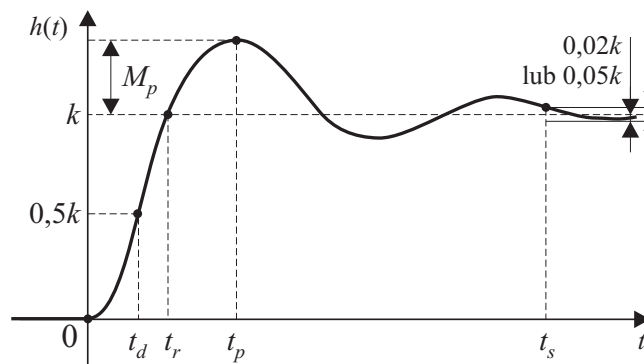
Rys. 58. zał. $T_i > T_d > T$, $\omega_y = \frac{1}{\sqrt{T_i T_d - T^2}}$



Rys. 59.

Własności regulatorów ciągłych

odpowieź układu zamkniętego	czas narastania t_r	przeregulowanie $M_p, \%$	czas ustalania t_s	uchyb ustalony e_u
$k_p \nearrow$	\searrow	\nearrow	$\approx \text{const}$	\searrow
$T_i \nearrow$	\nearrow	\searrow	\searrow	$\rightarrow 0$
$T_d \nearrow$	$\approx \text{const}$	\searrow	\searrow	$\approx \text{const}$



Rys. 60.

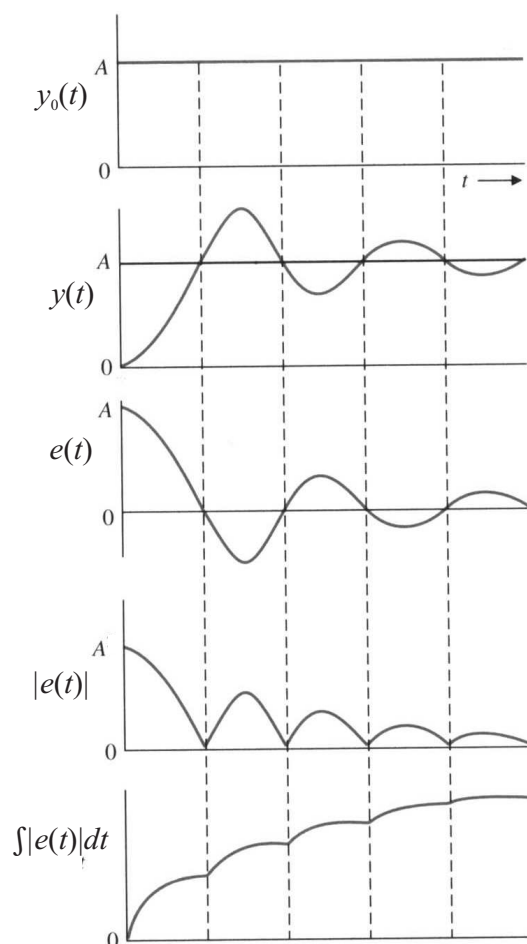
15. Całkowe wskaźniki jakości

Wskaźnik ISE (ang. *integral of the squared error*):

$$I_2 = \int_0^T e^2(t) dt \quad (53)$$

Wskaźnik IAE (ang. *integral of the absolute magnitude of the error*):

$$I_a = \int_0^T |e(t)| dt \quad (54)$$



Rys. 61.

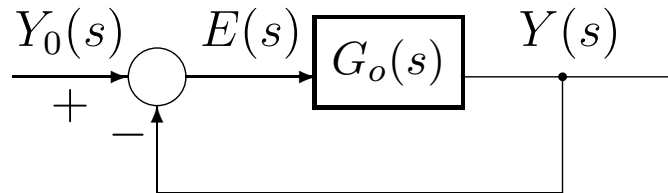
Wskaźnik ITAE (ang. *integral of time multiplied by the absolute magnitude of the error*):

$$I_{ta} = \int_0^T t|e(t)| dt \tag{55}$$

Wskaźnik ITSE (ang. *integral of time multiplied by the squared error*):

$$I_{t2} = \int_0^T te^2(t) dt \tag{56}$$

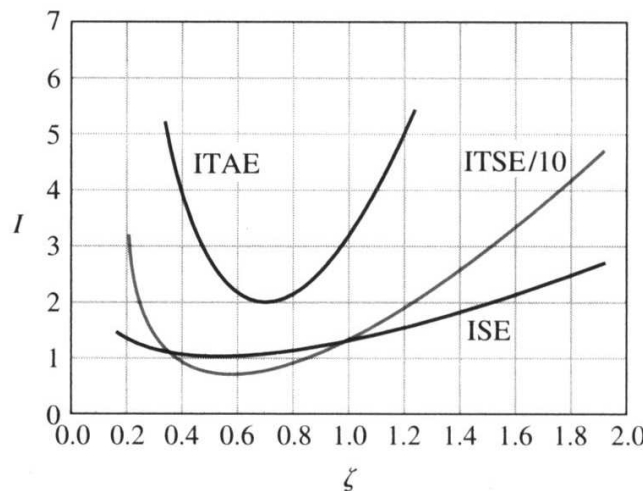
Przykład



Rys. 62.

$$G_o(s) = \frac{1}{s(s + 2\zeta)} \Rightarrow G(s) = \frac{G_o(s)}{1 + G_o(s)} = \frac{1}{s^2 + 2\zeta s + 1}$$

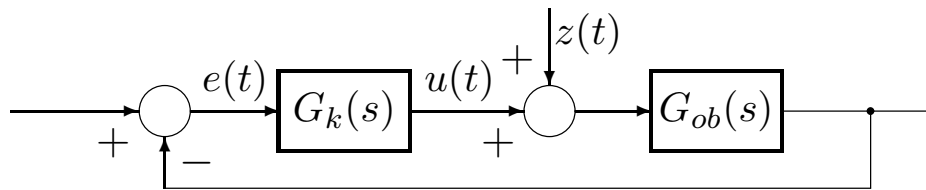
$$\omega_n = 1, \quad k = 1$$



Rys. 63.

16. Korekcja dynamiczna układów liniowych

a) korekcja szeregową

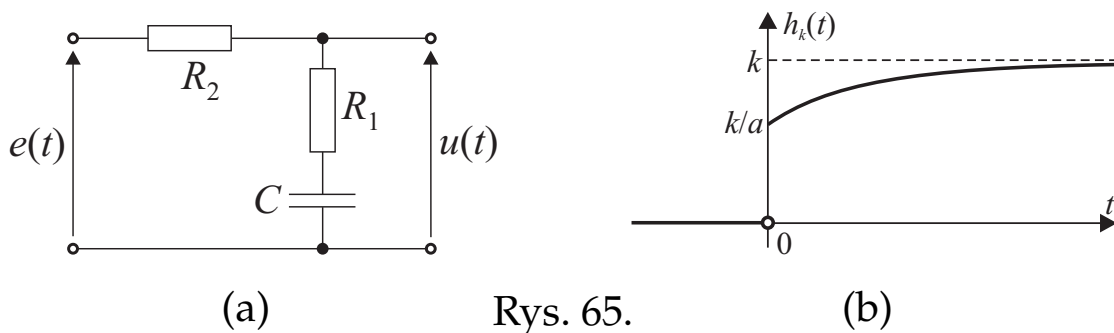


Rys. 64.

$$G_o(s) = G_k(s)G_{ob}(s), \quad G_k(s) = k \frac{1 + T_2s}{1 + T_1s} \quad (57)$$

$T_1 > T_2$ – k. opóźniająca fazę, $T_1 < T_2$ – k. przyspieszająca fazę

korektor opóźniająca fazę



Rys. 65.

$$\begin{aligned} G_k(s) &= \frac{U(s)}{E(s)} = \frac{R_1 + \frac{1}{sC}}{R_2 + (R_1 + \frac{1}{sC})} = \frac{1 + R_1Cs}{1 + (R_1 + R_2)Cs} = \quad (58) \\ &= k \frac{1 + T_2s}{1 + T_1s} = k \frac{1 + Ts}{1 + aTs}, \quad a > 1 \end{aligned}$$

$$k = 1, \quad T_1 = (R_1 + R_2)C, \quad T_2 = R_1C, \quad T = T_2, \quad a = \frac{R_1 + R_2}{R_1}$$

$$\begin{aligned} h_k(t) &= \mathcal{L}^{-1} \left\{ \frac{G_k(s)}{s} \right\} = \mathcal{L}^{-1} \left\{ \frac{k(1 + Ts)}{(1 + aTs)s} \right\} = \frac{k}{aT} \mathcal{L}^{-1} \left\{ \frac{1 + Ts}{(\frac{1}{aT} + s)s} \right\} = \\ &= \frac{k}{aT} \left(\frac{1 + T(-\frac{1}{aT})}{-\frac{1}{aT}} e^{-\frac{t}{aT}} + \frac{1}{\frac{1}{aT}} \right) \mathbb{1}(t) = k \left(1 - \frac{a-1}{a} e^{-\frac{t}{aT}} \right) \mathbb{1}(t) \end{aligned}$$

$$G_k(s) = k \frac{1 + Ts}{1 + aTs}, \quad a > 1 \rightarrow G_k(j\omega) = k \frac{1 + j\omega T}{1 + j\omega aT}$$

$$aT > T \rightarrow 1/(aT) < 1/T$$

$$Lm_k(\omega) = 20 \log k + 20 \log \sqrt{1 + \omega^2 T^2} - 20 \log \sqrt{1 + \omega^2 a^2 T^2} \approx$$

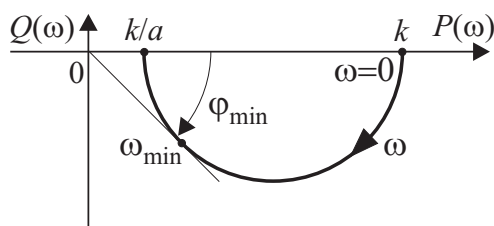
$$\approx \begin{cases} 20 \log k, & \omega < 1/(aT) \\ 20 \log k - 20 \log(\omega aT), & 1/(aT) \leq \omega < 1/T \\ \underbrace{20 \log k + 20 \log(\omega T) - 20 \log(\omega aT)}_{=20 \log(k/a)}, & \omega \geq 1/T \end{cases}$$

$$\varphi_k(\omega) = \arctg(\omega T) - \arctg(\omega aT)$$

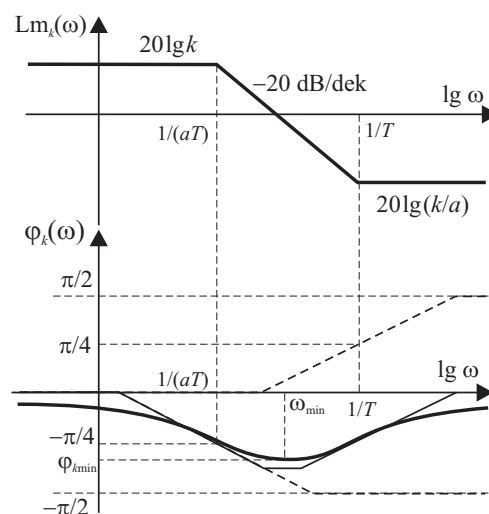
$$\frac{d\varphi_k(\omega)}{d\omega} = \frac{T}{1 + \omega^2 T^2} - \frac{aT}{1 + \omega^2 a^2 T^2} = \frac{T(1 + \omega^2 a^2 T^2) - aT(1 + \omega^2 T^2)}{(1 + \omega^2 T^2)(1 + \omega^2 a^2 T^2)}$$

$$1 - a + \omega^2 T^2 (a^2 - a) = 0 \rightarrow 1 - \omega^2 T^2 a = 0 \rightarrow \omega_{\min} = \frac{1}{T\sqrt{a}} = \frac{1}{\sqrt{T_1 T_2}}$$

$$\varphi_{k \min} = \arctg \frac{1}{\sqrt{a}} - \arctg \sqrt{a}$$



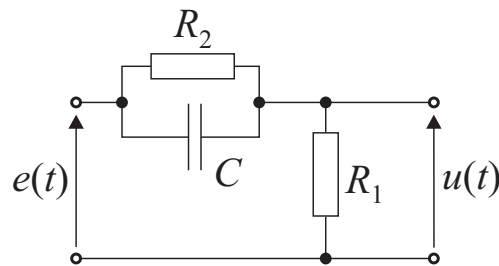
(a)



(b)

Rys. 66.

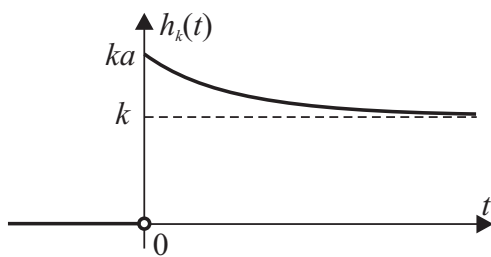
korektor przyspieszający fazę



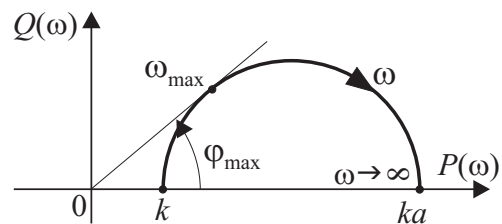
Rys. 67.

$$G_k(s) = \frac{U(s)}{E(s)} = k \frac{1 + T_2 s}{1 + T_1 s} = \frac{1}{a} \frac{1 + a T s}{1 + T s}, \quad a > 1 \quad (59)$$

$$T_1 = \frac{C R_1 R_2}{R_1 + R_2}, \quad T_2 = R_2 C, \quad k = \frac{R_1}{R_1 + R_2}, \quad T = T_2, \quad a = \frac{1}{k}$$



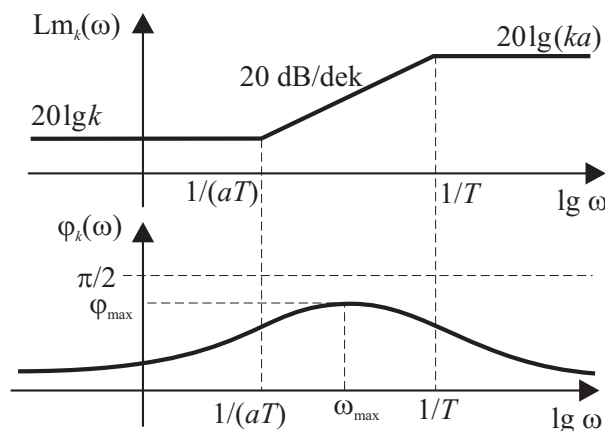
(a)



(b)

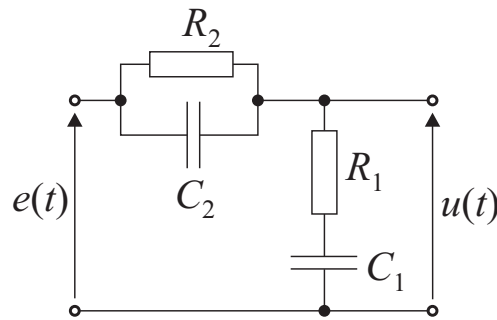
Rys. 68. $\omega_{\max} = \frac{1}{T\sqrt{a}} = \frac{1}{\sqrt{T_1 T_2}}$

$$a > 1 \rightarrow aT > T \rightarrow 1/(aT) < 1/T$$



Rys. 69.

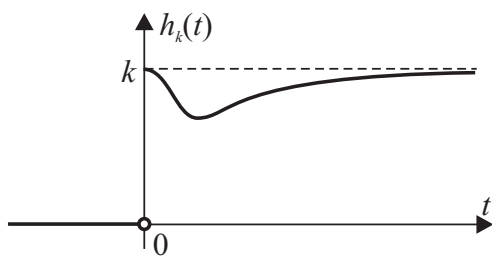
korektor przyspieszająco-opóźniający



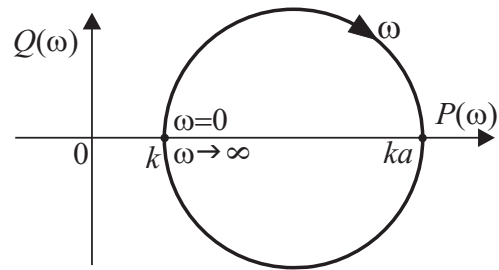
Rys. 70.

$$G_k(s) = k \frac{(1 + T_2s)(1 + T_4s)}{(1 + T_1s)(1 + T_3s)} = k \frac{1 + aT_1s}{1 + T_1s} \frac{1 + T_4s}{1 + aT_4s}, \quad (60)$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = a > 1, \quad T_3 > T_4 > T_2 > T_1$$



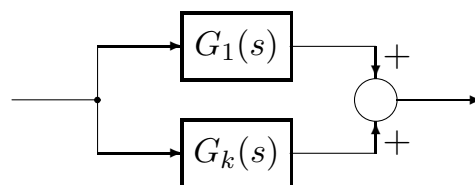
(a)



(b)

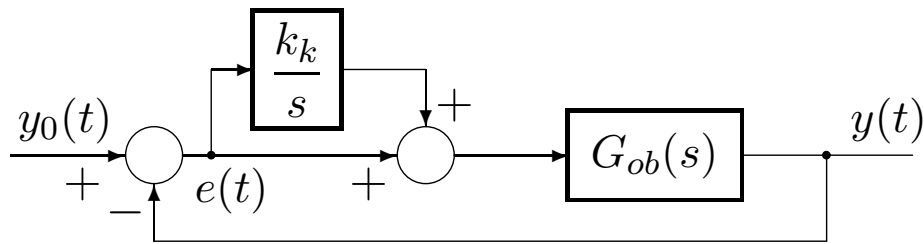
Rys. 71.

b) korekcja równoległa



Rys. 72.

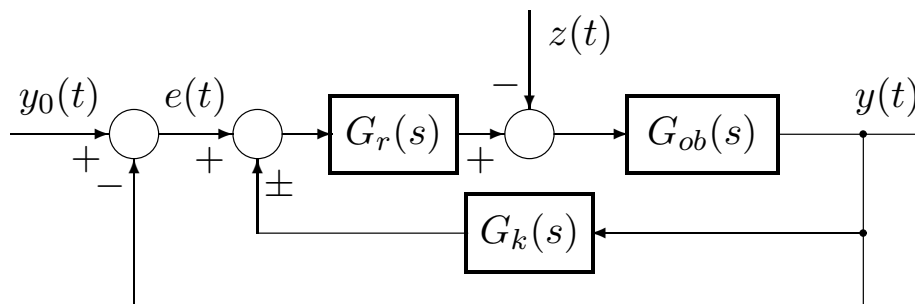
$$G(s) = G_1(s) + G_k(s)$$



Rys. 73.

$$G_o(s) = \frac{Y(s)}{E(s)} = \left(1 + \frac{k_k}{s}\right) G_{ob}(s) = \left(1 + \frac{1}{T_k s}\right) G_{ob}(s)$$

c) korekcja ze sprzężeniem zwrotnym



Rys. 74.

$$G_{ok}(s) = \frac{Y(s)}{E(s)} = \frac{G_r(s)G_{ob}(s)}{1 \mp G_k(s)G_r(s)G_{ob}(s)}$$

odp. skokowa układu otwartego ze sprzężeniem korekcyjnym:

$$h_{ok}(t) = \mathcal{L}^{-1} \left\{ G_{ok}(s) \frac{1}{s} \right\} = \mathcal{L}^{-1} \left\{ \frac{G_r(s)G_{ob}(s)}{(1 \mp G_k(s)G_r(s)G_{ob}(s))s} \right\}$$

odp. skokowa układu otwartego bez sprzężenia korekcyjnego:

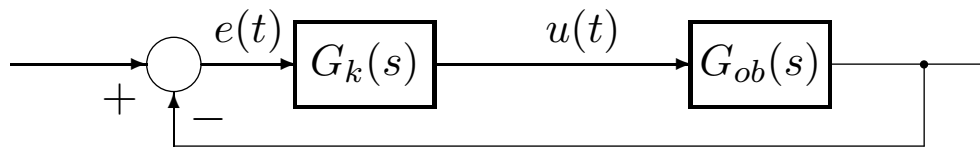
$$h_o(t) = \mathcal{L}^{-1} \left\{ G_o(s) \frac{1}{s} \right\} = \mathcal{L}^{-1} \left\{ \frac{G_r(s)G_{ob}(s)}{s} \right\}$$

sprężenie elastyczne $\Rightarrow \lim_{t \rightarrow \infty} h_{ok}(t) = \lim_{t \rightarrow \infty} h_o(t)$, tzn.

$$\lim_{s \rightarrow 0} s \frac{G_r(s)G_{ob}(s)}{(1 \mp G_k(s)G_r(s)G_{ob}(s))s} = \lim_{s \rightarrow 0} s \frac{G_r(s)G_{ob}(s)}{s}$$

$$\lim_{s \rightarrow 0} G_k(s)G_r(s)G_{ob}(s) = 0$$

Przykład (korektor szeregowy opóźniający fazę)



Rys. 75.

$$G_{ob}(s) = \frac{k}{(T_1s + 1)(T_2s + 1)(T_3s + 1)}, \quad G_k(s) = \frac{1 + Ts}{1 + aTs}, \quad a > 1$$

$$k = 100, \quad T_1 = 0,2[s], \quad T_2 = 0,4[s], \quad T_3 = 2,5[s]$$

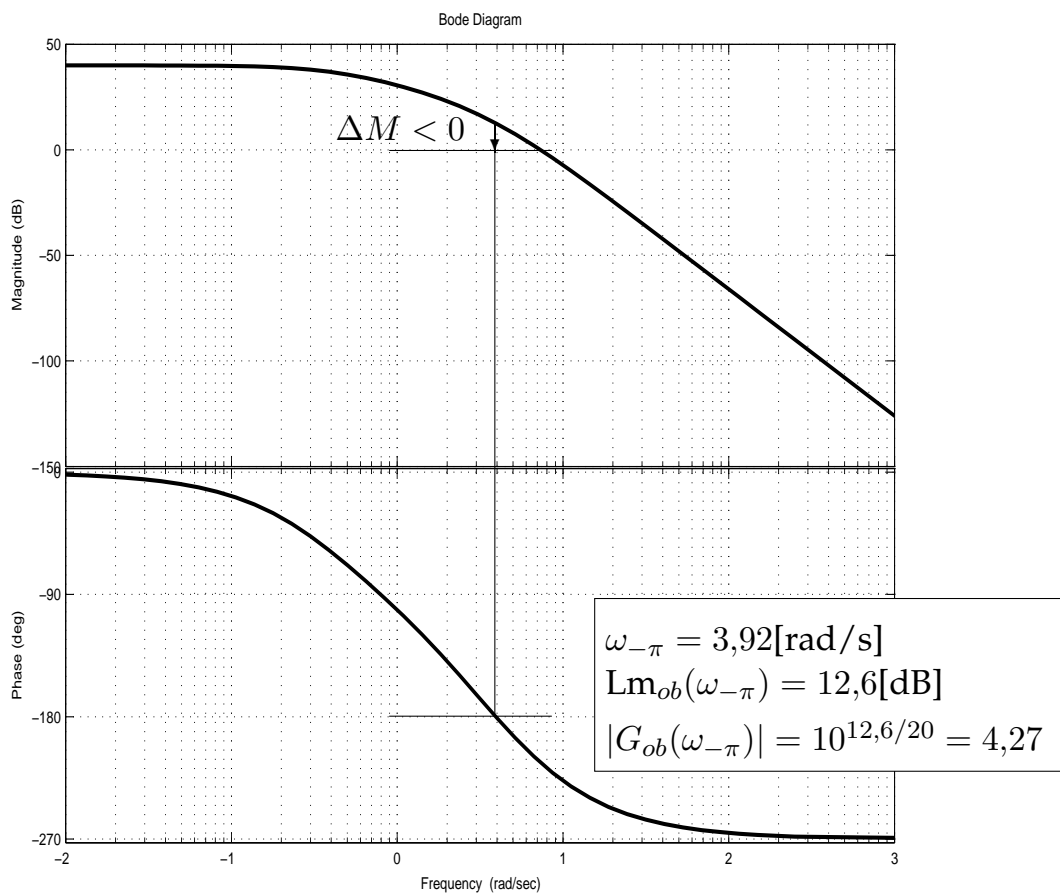
$$k=100; \quad T1=0.2; \quad T2=0.4; \quad T3=2.5;$$

$$G_ob=tf(k,conv(conv([T1 \ 1],[T2 \ 1]),[T3 \ 1]));$$

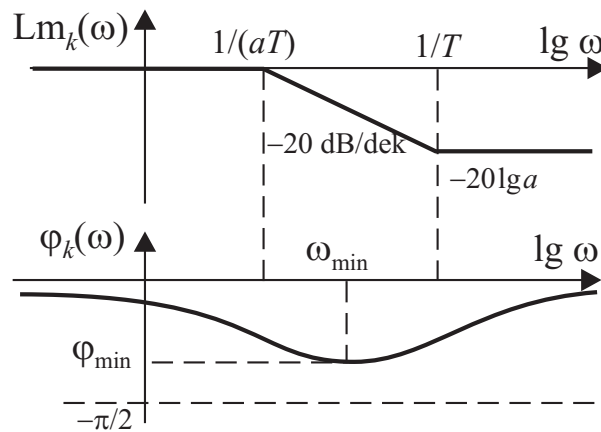
$$bode(G_ob); \quad \text{grid on}$$

LTI (Linear Time-Invariant) Viewer:

$$\text{ltiview('bode', G_ob)}$$



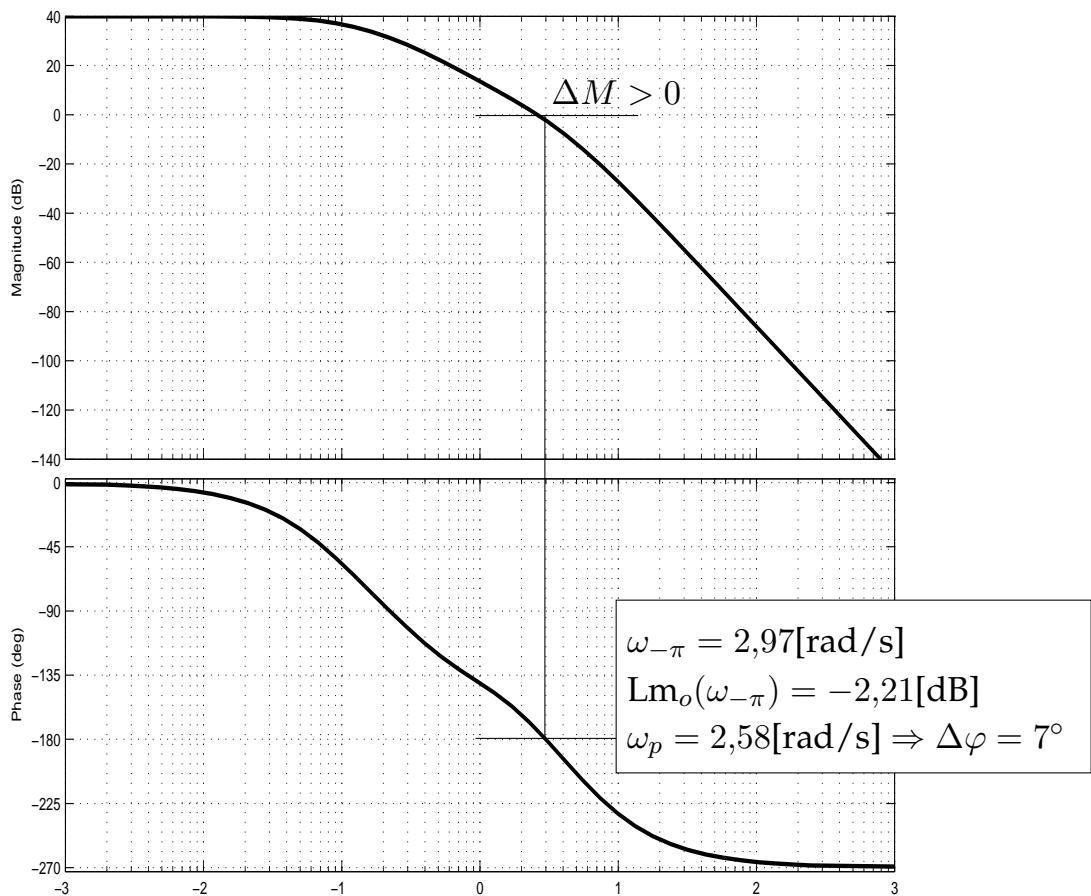
Rys. 76.



Rys. 77.

$T \gg 1/\omega_{-\pi} = 0,26[s], a > |G_{ob}(\omega_{-\pi})| = 4,27: T = 1[s], a = 10$

```
T = 1; a = 10;
G_k = tf([T 1],[a*T 1]);
G_o = series(G_k,G_ob);
bode(G_o); grid on
```



Rys. 78. $e_u = 1/(1 + 100) \approx 0,01$