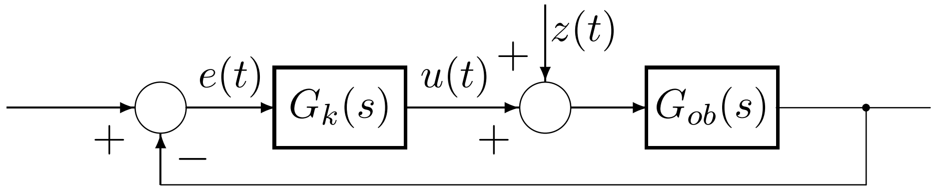
36. Korekcja dynamiczna układów liniowych

a) korekcja szeregowa

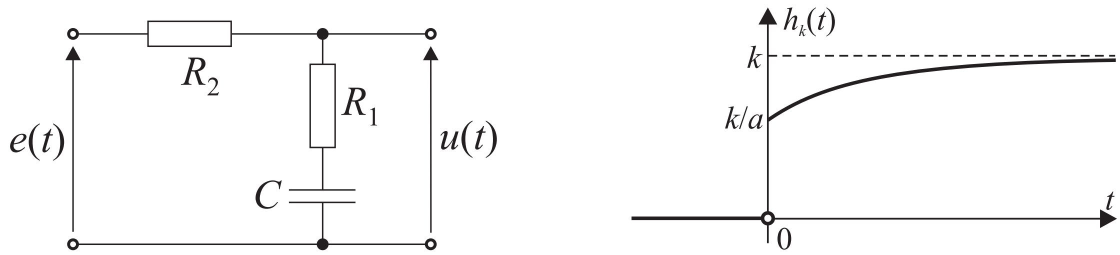


Rys. 111.

$$G_o(s) = G_k(s)G_{ob}(s), \qquad G_k(s) = k\frac{1+T_2 s}{1+T_1 s} \tag{109}$$

$T_1 > T_2$ – k. opóźniający fazę, $\quad T_1 < T_2$ – k. przyspieszający fazę

korektor opóźniający fazę



(a) Rys. 112. (b)

$$G_k(s) = \frac{U(s)}{E(s)} = \frac{R_1 + \frac{1}{sC}}{R_2 + (R_1 + \frac{1}{sC})} = \frac{1 + R_1 C s}{1 + (R_1 + R_2)Cs} = \tag{110}$$
$$= k\frac{1+T_2 s}{1+T_1 s} = k\frac{1+Ts}{1+aTs}, \quad a > 1$$

$$k = 1, \quad T_1 = (R_1 + R_2)C, \quad T_2 = R_1 C, \qquad T = T_2, \quad a = \frac{R_1 + R_2}{R_1}$$

$$h_k(t) = \mathcal{L}^{-1}\left\{\frac{G_k(s)}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{k(1+Ts)}{(1+aTs)s}\right\} = \frac{k}{aT}\mathcal{L}^{-1}\left\{\frac{1+Ts}{(\frac{1}{aT}+s)s}\right\} =$$
$$= \frac{k}{aT}\left(\frac{1+T(-\frac{1}{aT})}{-\frac{1}{aT}}e^{-\frac{t}{aT}} + \frac{1}{\frac{1}{aT}}\right)\mathbb{1}(t) = k\left(1 - \frac{a-1}{a}e^{-\frac{t}{aT}}\right)\mathbb{1}(t)$$

$$G_k(s) = k\frac{1+Ts}{1+aTs}, \quad a>1 \quad \rightarrow \quad G_k(j\omega) = k\frac{1+j\omega T}{1+j\omega aT}$$

$$aT > T \quad \rightarrow \quad 1/(aT) < 1/T$$

$$\mathrm{Lm}_k(\omega) = 20\log k + 20\log\sqrt{1+\omega^2T^2} - 20\log\sqrt{1+\omega^2a^2T^2} \approx$$

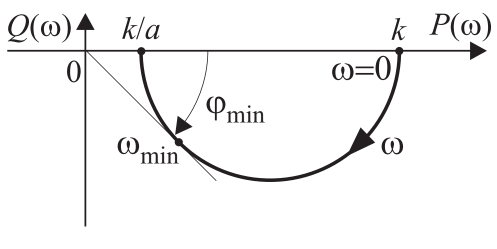
$$\approx \begin{cases} 20\log k, & \omega < 1/(aT) \\ 20\log k - 20\log(\omega aT), & 1/(aT) \leqslant \omega < 1/T \\ \underbrace{20\log k + 20\log(\omega T) - 20\log(\omega aT)}_{=20\log(k/a)}, & \omega \geqslant 1/T \end{cases}$$

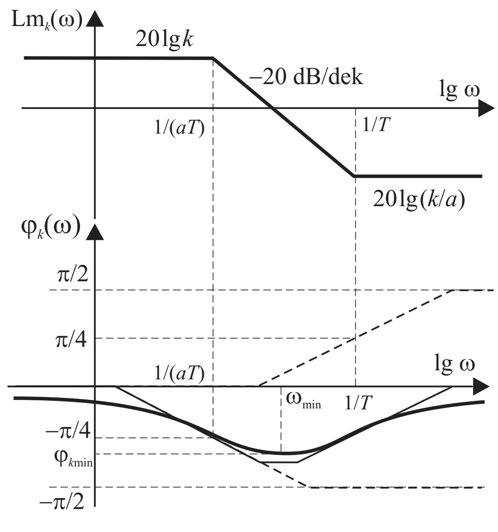
$$\varphi_k(\omega) = \mathrm{arctg}(\omega T) - \mathrm{arctg}(\omega aT)$$

$$\frac{d\varphi_k(\omega)}{d\omega} = \frac{T}{1+\omega^2T^2} - \frac{aT}{1+\omega^2a^2T^2} = \frac{T(1+\omega^2a^2T^2) - aT(1+\omega^2T^2)}{(1+\omega^2T^2)(1+\omega^2a^2T^2)}$$

$$1-a+\omega^2T^2(a^2-a)=0 \quad \rightarrow \quad 1-\omega^2T^2a=0 \quad \rightarrow \quad \omega_{\min} = \frac{1}{T\sqrt{a}} = \frac{1}{\sqrt{T_1T_2}}$$

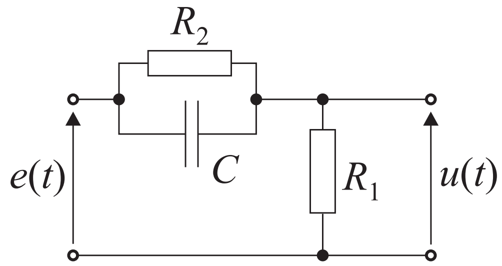
$$\varphi_{k\,\min} = \mathrm{arctg}\frac{1}{\sqrt{a}} - \mathrm{arctg}\sqrt{a}$$





(a) Rys. 113. (b)

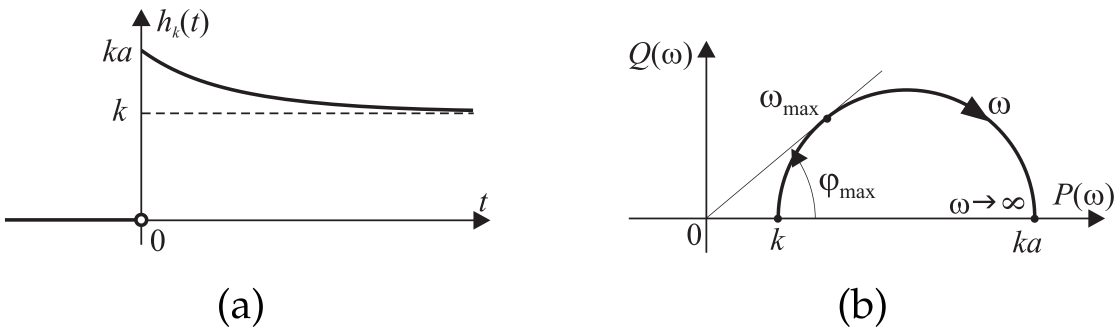
korektor przyspieszający fazę



Rys. 114.

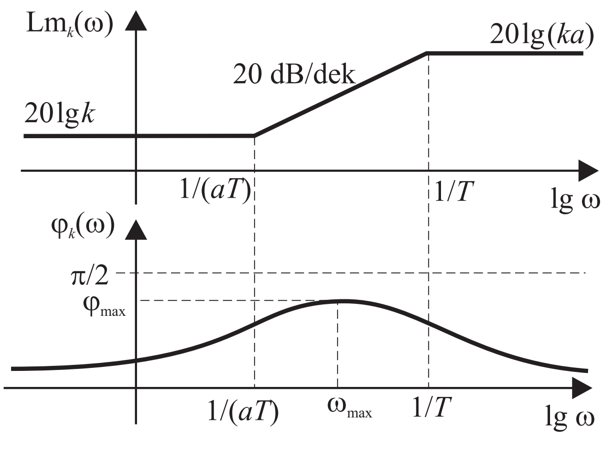
$$G_k(s) = \frac{U(s)}{E(s)} = k\frac{1+T_2 s}{1+T_1 s} = \frac{1}{a}\frac{1+aTs}{1+Ts}, \quad a>1 \qquad (111)$$

$$T_1 = \frac{CR_1R_2}{R_1+R_2}, \quad T_2 = R_2C, \quad k = \frac{R_1}{R_1+R_2}, \quad T = T_2, \quad a = \frac{1}{k}$$



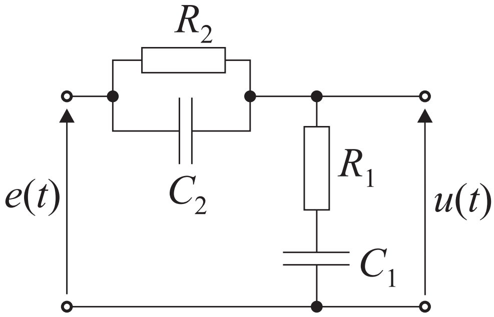
Rys. 115. $\omega_{\max} = \frac{1}{T\sqrt{a}} = \frac{1}{\sqrt{T_1T_2}}$

$$a>1 \;\rightarrow\; aT>T \;\rightarrow\; 1/(aT) < 1/T$$



Rys. 116.

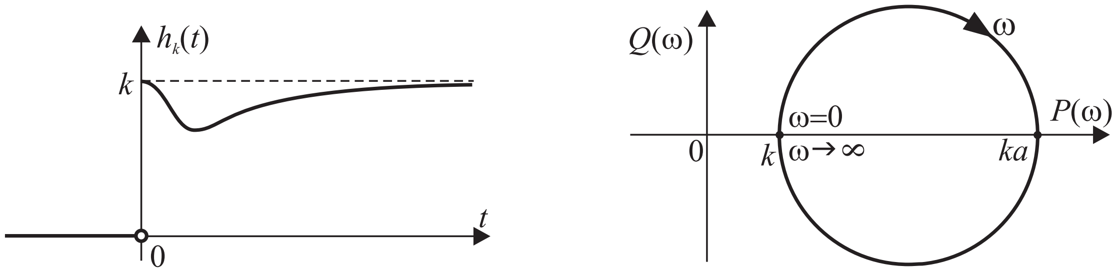
korektor przyspieszająco-opóźniający



Rys. 117.

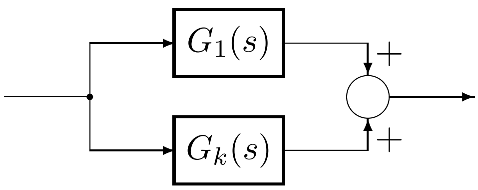
$$G_k(s) = k\frac{(1+T_2 s)(1+T_4 s)}{(1+T_1 s)(1+T_3 s)} = k\frac{1+aT_1 s}{1+T_1 s}\frac{1+T_4 s}{1+aT_4 s}, \qquad (112)$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = a > 1, \qquad T_3 > T_4 > T_2 > T_1$$



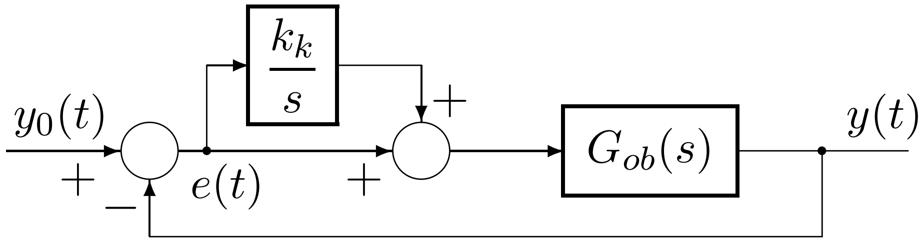
(a) Rys. 118. (b)

b) korekcja równoległa



Rys. 119.

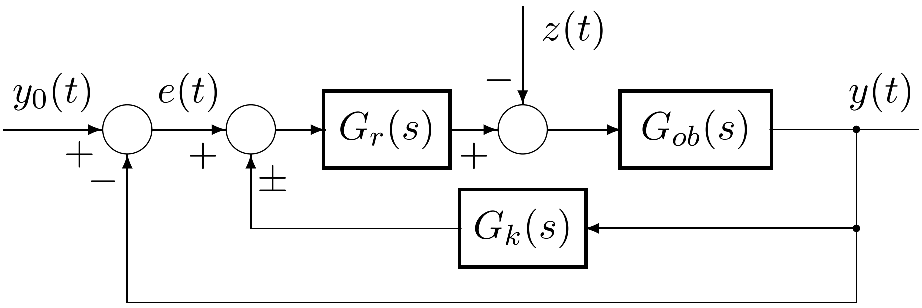
$$G(s) = G_1(s) + G_k(s)$$



Rys. 120.

$$G_o(s) = \frac{Y(s)}{E(s)} = \left(1 + \frac{k_k}{s}\right) G_{ob}(s) = \left(1 + \frac{1}{T_k s}\right) G_{ob}(s)$$

c) korekcja ze sprzężeniem zwrotnym



Rys. 121.

$$G_{ok}(s) = \frac{Y(s)}{E(s)} = \frac{G_r(s)G_{ob}(s)}{1 \mp G_k(s)G_r(s)G_{ob}(s)}$$

odp. skokowa układu otwartego ze sprzężeniem korekcyjnym:

$$h_{ok}(t) = \mathcal{L}^{-1}\left\{G_{ok}(s)\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{G_r(s)G_{ob}(s)}{[1 \mp G_k(s)G_r(s)G_{ob}(s)]s}\right\}$$

odp. skokowa układu otwartego bez sprzężenia korekcyjnego:

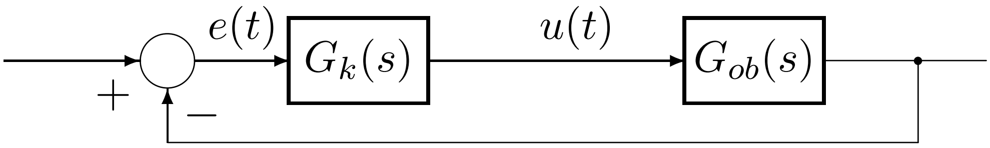
$$h_o(t) = \mathcal{L}^{-1}\left\{G_o(s)\frac{1}{s}\right\} = \mathcal{L}^{-1}\left\{\frac{G_r(s)G_{ob}(s)}{s}\right\}$$

sprzężenie elastyczne $\Rightarrow \lim\limits_{t\to\infty} h_{ok}(t) = \lim\limits_{t\to\infty} h_o(t)$, tzn.

$$\lim_{s\to 0} s\frac{G_r(s)G_{ob}(s)}{[1 \mp G_k(s)G_r(s)G_{ob}(s)]s} = \lim_{s\to 0} s\frac{G_r(s)G_{ob}(s)}{s}$$

$$\lim_{s\to 0} G_k(s)G_r(s)G_{ob}(s) = 0$$

Przykład (korektor szeregowy opóźniający fazę)



Rys. 122.

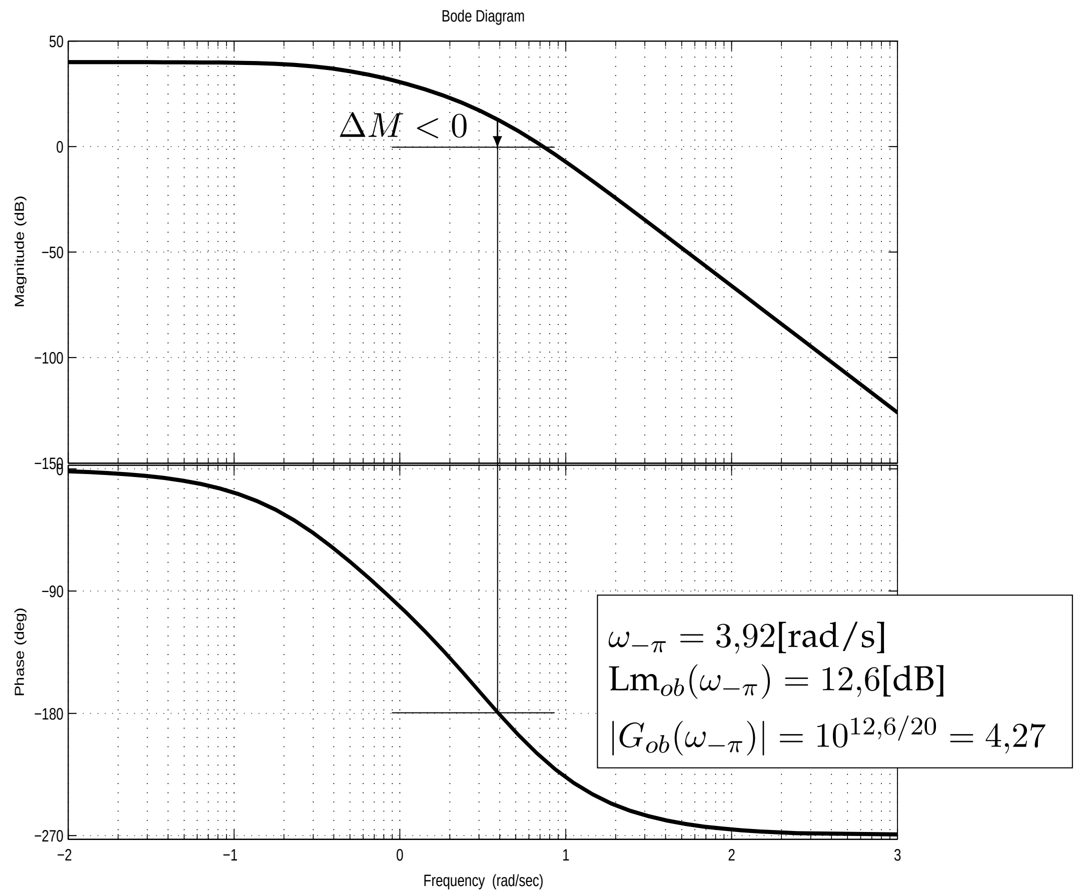
$$G_{ob}(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)}, \quad G_k(s) = \frac{1 + Ts}{1 + aTs}, \quad a > 1$$

$$k = 100, \quad T_1 = 0{,}2[\mathrm{s}], \quad T_2 = 0{,}4[\mathrm{s}], \quad T_3 = 2{,}5[\mathrm{s}]$$

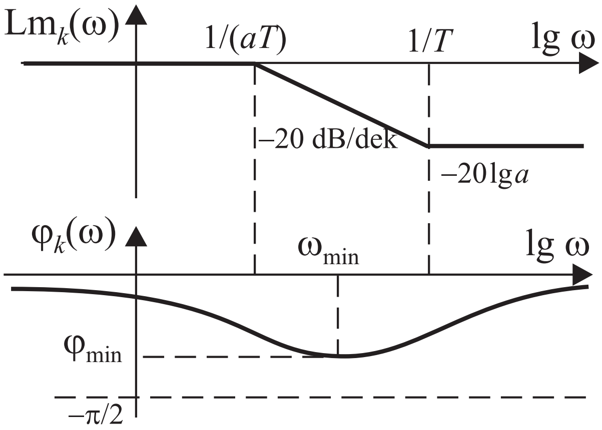
```
k=100; T1=0.2; T2=0.4; T3=2.5;
G_ob=tf(k,conv(conv([T1 1],[T2 1]),[T3 1]));
bode(G_ob); grid on
```

LTI (Linear Time-Invariant) Viewer:

```
ltiview('bode',G_ob)
```



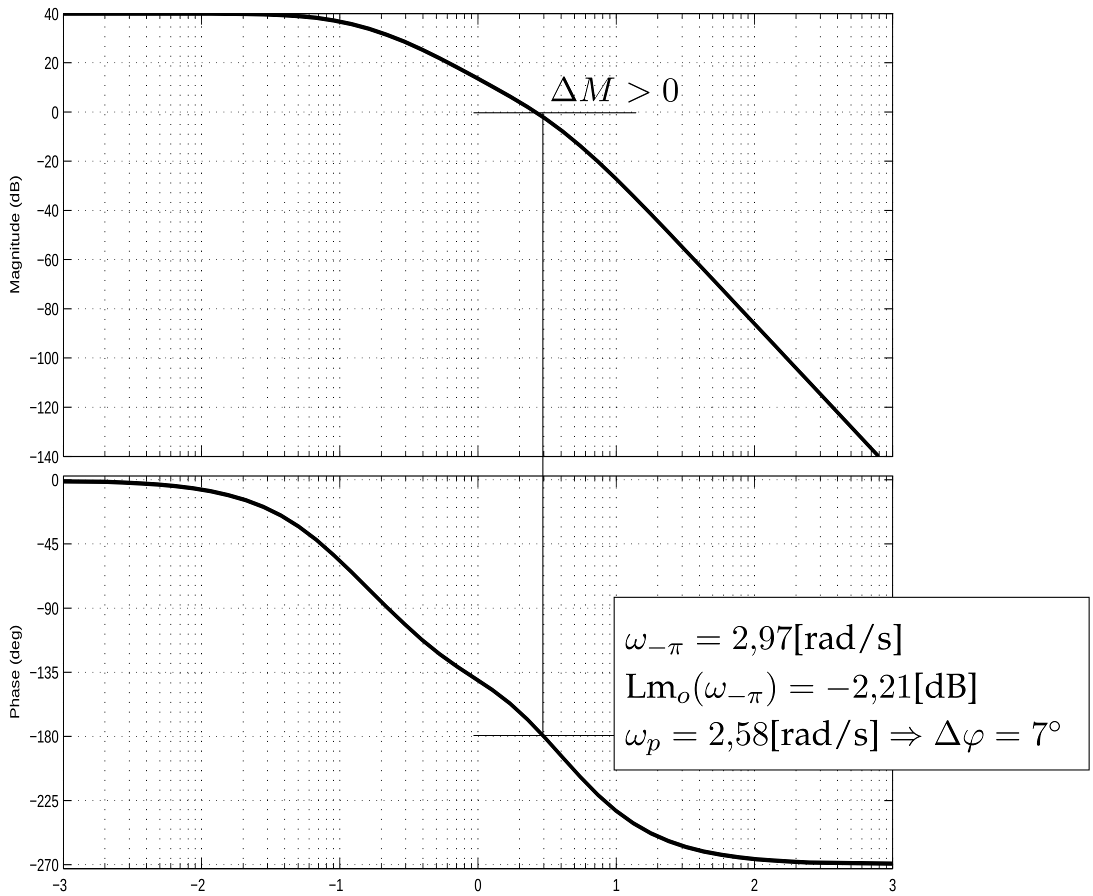
Rys. 123.



Rys. 124.

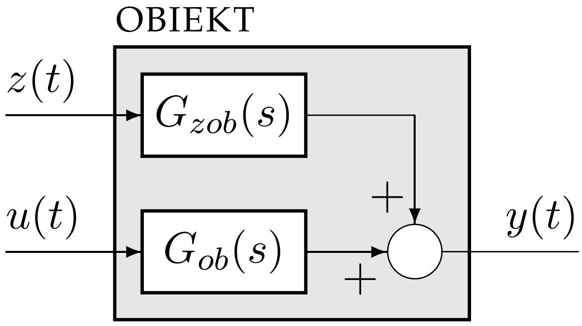
$T \gg 1/\omega_{-\pi} = 0{,}26$[s], $a > |G_{ob}(\omega_{-\pi})| = 4{,}27$: $T = 1$[s], $a = 10$

```
T = 1; a = 10;
G_k = tf([T 1],[a*T 1]);
G_o = series(G_k,G_ob);
bode(G_o); grid on
```

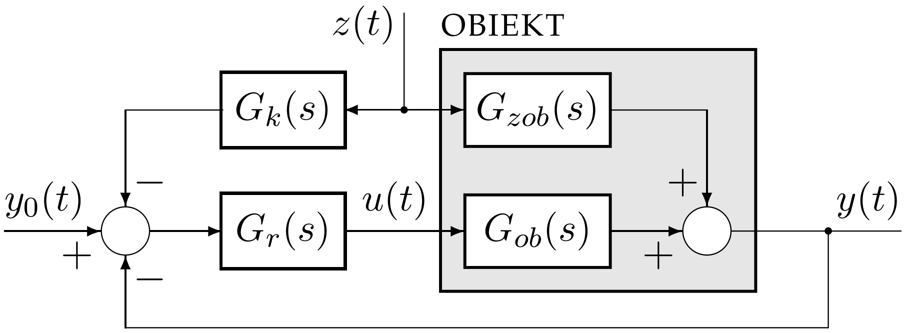


Rys. 125. $e_u = 1/(1 + 100) \approx 0{,}01$

37. Sprzężenie „w przód" (ang. *feedforward*)



Rys. 126.



Rys. 127.

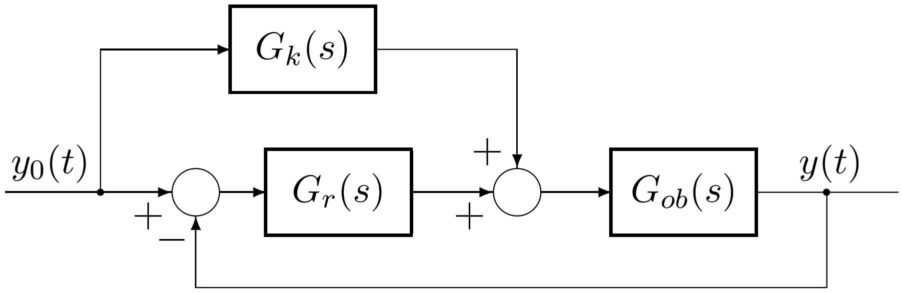
$$Y(s)\Big|_{y_0(t)\equiv 0} = \frac{G_{zob}(s)}{1+G_r(s)G_{ob}(s)}Z(s) - \frac{G_k(s)G_r(s)G_{ob}(s)}{1+G_r(s)G_{ob}(s)}Z(s) \quad (113)$$

$$G_z(s) = \frac{Y(s)}{Z(s)} = \frac{G_{zob}(s) - G_k(s)G_r(s)G_{ob}(s)}{1+G_r(s)G_{ob}(s)} \quad (114)$$

w przypadku idealnym:

$$G_{zob}(s) - G_k(s)G_r(s)G_{ob}(s) = 0$$

$$G_k(s)G_r(s) = \frac{G_{zob}(s)}{G_{ob}(s)} \quad (115)$$



Rys. 128.

$$G(s)=\frac{Y(s)}{Y_0(s)}=\left(1+\frac{G_k(s)}{G_r(s)}\right)\frac{G_r(s)G_{ob}(s)}{1+G_r(s)G_{ob}(s)}=\frac{[G_k(s)+G_r(s)]G_{ob}(s)}{1+G_r(s)G_{ob}(s)} \tag{116}$$

$$G_k(s)=\frac{L_k(s)}{M_k(s)},\qquad G_r(s)=\frac{L_r(s)}{M_r(s)},\qquad G_{ob}(s)=\frac{L_{ob}(s)}{M_{ob}(s)}$$

$$G(s)=\frac{\left(\frac{L_r(s)}{M_r(s)}+\frac{L_k(s)}{M_k(s)}\right)\frac{L_{ob}(s)}{M_{ob}(s)}}{1+\frac{L_r(s)}{M_r(s)}\frac{L_{ob}(s)}{M_{ob}(s)}}=\frac{\frac{[L_r(s)M_k(s)+L_k(s)M_r(s)]L_{ob}(s)}{M_k(s)M_r(s)M_{ob}(s)}}{\frac{L_r(s)L_{ob}(s)+M_r(s)M_{ob}(s)}{M_r(s)M_{ob}(s)}}=$$

$$=\frac{[L_r(s)M_k(s)+L_k(s)M_r(s)]L_{ob}(s)}{M_k(s)[L_r(s)L_{ob}(s)+M_r(s)M_{ob}(s)]} \tag{117}$$

równanie charakterystyczne:

$$M_k(s)[L_r(s)L_{ob}(s)+M_r(s)M_{ob}(s)]=0 \tag{118}$$

$$\frac{G_r(s)G_{ob}(s)}{1+G_r(s)G_{ob}(s)},\qquad G_k(s)$$

$$G_k(s)=\frac{1}{G_{ob}(s)}=\frac{M_{ob}(s)}{L_{ob}(s)}\quad\Rightarrow\quad G(s)=1 \tag{119}$$