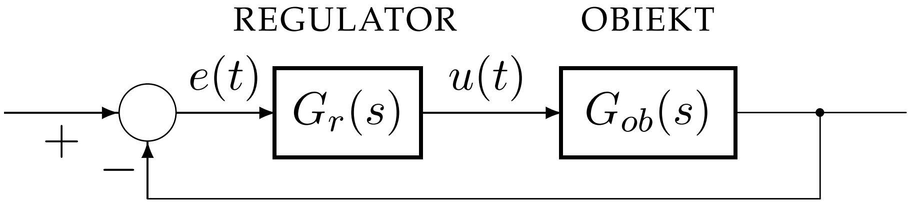
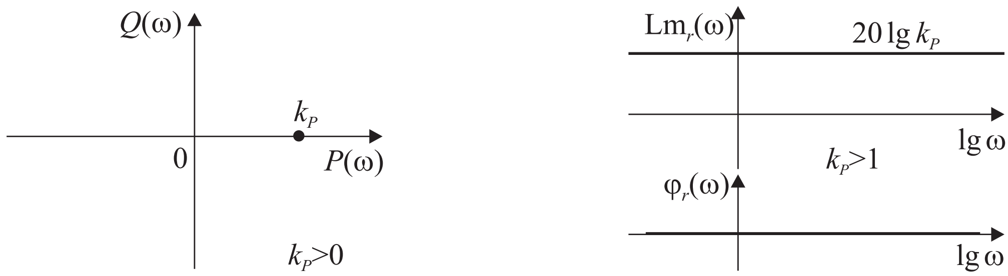
27. Regulatory liniowe o wyjściu ciągłym



Rys. 77.

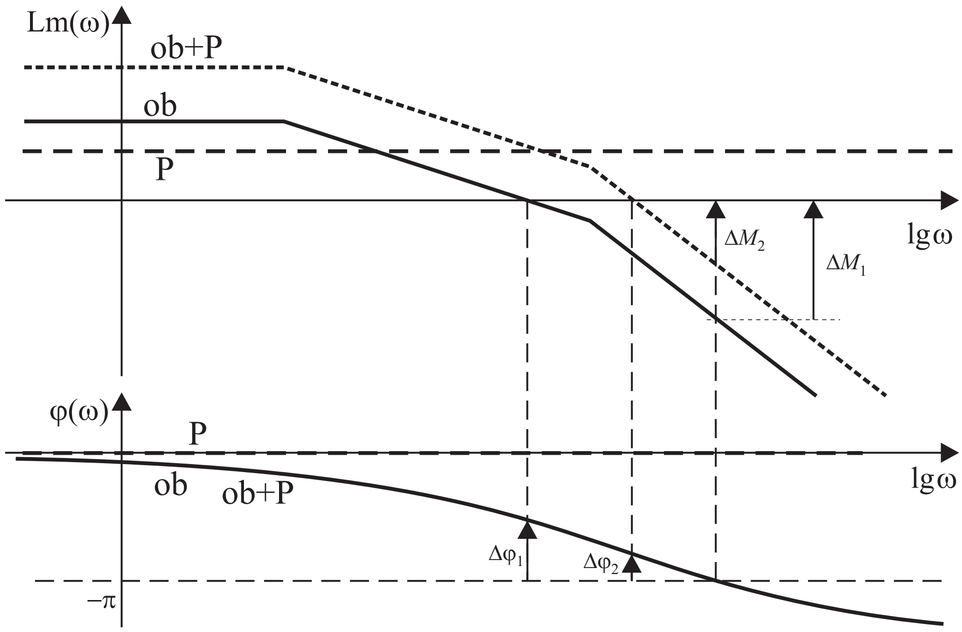
a) regulator typu P (proporcjonalny):

$$G_r(s) = \frac{U(s)}{E(s)} = k_p, \quad G_r(j\omega) = G_r(s)|_{s=j\omega} = k_p + j0, \quad k_p > 0$$



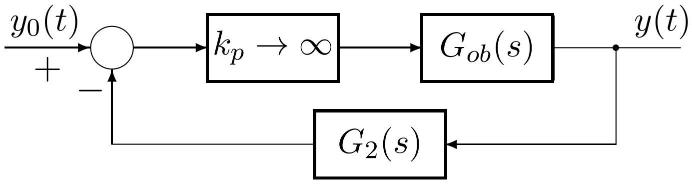
(a) (b)

Rys. 78.



Rys. 79.

$$e_u = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1 + k_p k_{ob}}$$



Rys. 80.

$$G(s) = \frac{Y(s)}{Y_0(s)} = \frac{k_p G_{ob}(s)}{1 + k_p G_{ob}(s) G_2(s)}$$

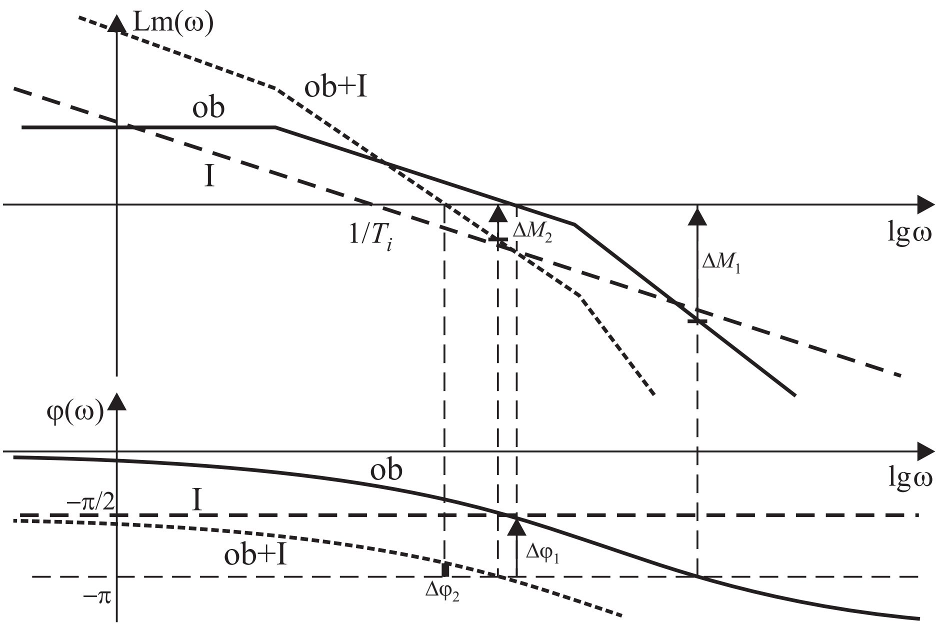
$$\lim_{k_p \to \infty} G(s) = \lim_{k_p \to \infty} \frac{G_{ob}(s)}{1/k_p + G_{ob}(s) G_2(s)} = \frac{1}{G_2(s)} \qquad (85)$$

b) regulator typu I (całkujący):

$$G_r(s) = \frac{U(s)}{E(s)} = \frac{1}{T_i s}, \qquad G_r(j\omega) = G_r(s)\big|_{s=j\omega} = \frac{1}{jT_i\omega} = 0 - j\frac{1}{T_i\omega}$$

$$\mathrm{Lm}_r(\omega) = -20\log(T_i) - 20\log(\omega), \qquad \varphi_r(\omega) = -\pi/2$$

(a) Rys. 81. (b)

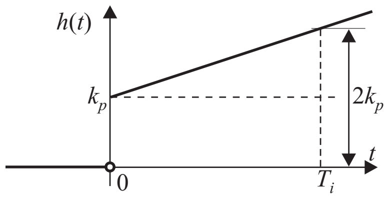


Rys. 82.

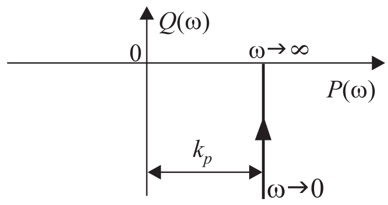
c) regulator typu PI (proporcjonalno-całkujący)

$$G_r(s) = \frac{U(s)}{E(s)} = k_p\left(1 + \frac{1}{T_i s}\right) \;\Rightarrow\; u(t) = k_p\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right) \quad (86)$$

$$h(t) = \mathcal{L}^{-1}\left\{G_r(s)\frac{1}{s}\right\} = k_p\left(1 + \frac{t}{T_i}\right)\mathbb{1}(t) \quad (87)$$



(a)

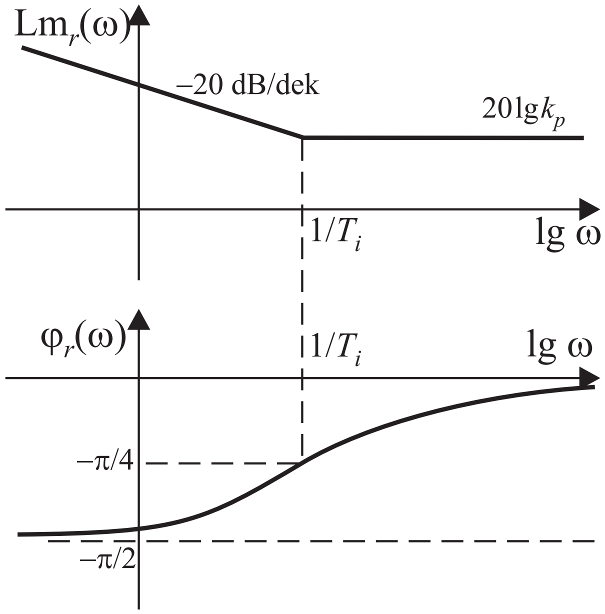


(b)

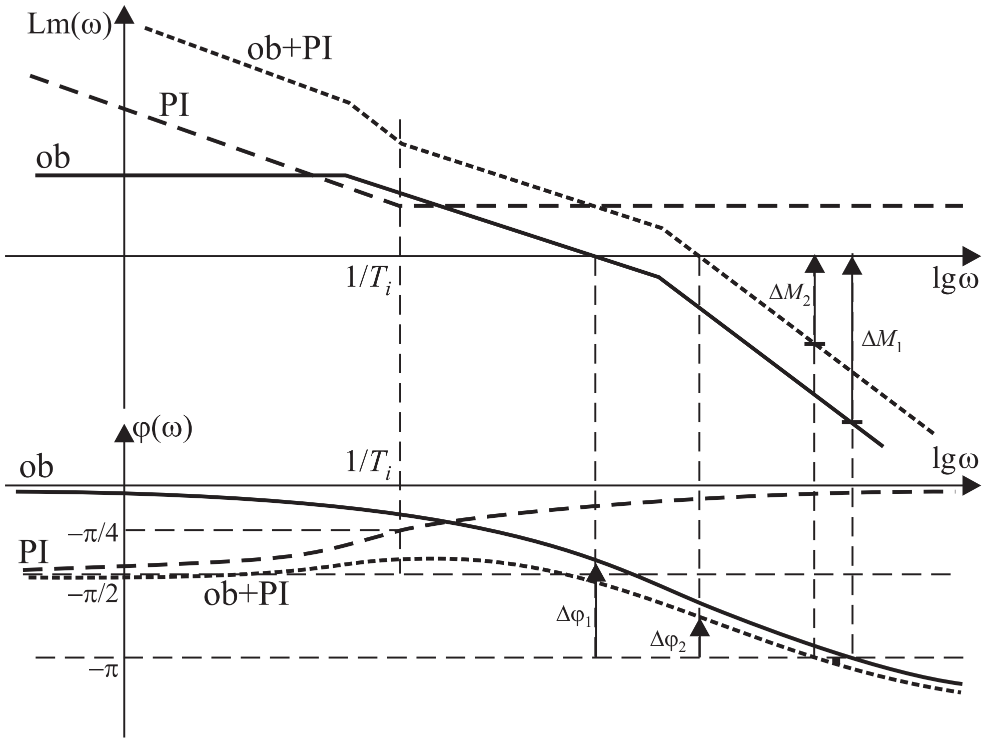
Rys. 83.

$$G_r(j\omega) = k_p\left(1 + \frac{1}{jT_i\omega}\right) = k_p\left(1 - j\frac{1}{T_i\omega}\right) \quad \Rightarrow \quad \varphi_r(\omega) = -\mathrm{arctg}\frac{1}{T_i\omega}$$

$$\mathrm{Lm}_r(j\omega) = 20\lg\left(k_p\sqrt{1 + \frac{1}{T_i^2\omega^2}}\right) \approx \begin{cases} 20\lg k_p - 20\lg(T_i\omega), & \omega < 1/T_i \\ 20\lg k_p, & \omega \geqslant 1/T_i \end{cases}$$



Rys. 84.



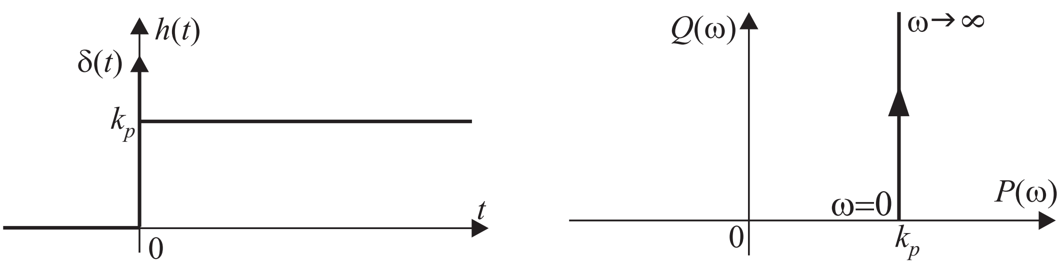
Rys. 85.

d) regulator typu PD (proporcjonalno-różniczkujący)

∗ regulator typu PD idealny

$$G_r(s) = \frac{U(s)}{E(s)} = k_p(1 + T_d s) \quad \Rightarrow \quad u(t) = k_p\left(e(t) + T_d\frac{de(t)}{dt}\right) \tag{88}$$

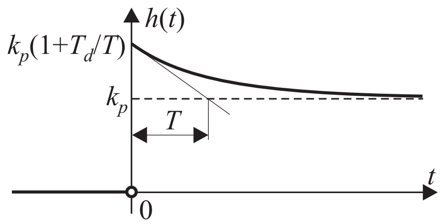
$$h(t) = k_p\big(\mathbb{1}(t) + T_d\delta(t)\big), \qquad G_r(j\omega) = G_r(s)|_{s=j\omega} = k_p(1 + jT_d\omega)$$



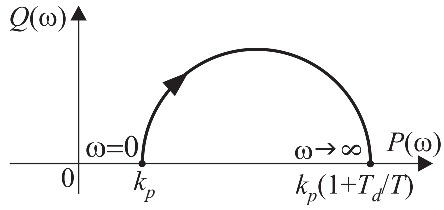
(a) Rys. 86. (b)

∗ regulator typu PD rzeczywisty

$$G_r(s) = \frac{U(s)}{E(s)} = k_p\left(1 + \frac{T_d s}{1+Ts}\right) \Rightarrow h(t) = k_p\left(1 + \frac{T_d}{T}e^{-\frac{t}{T}}\right)\mathbb{1}(t) \quad (89)$$



(a)



(b)

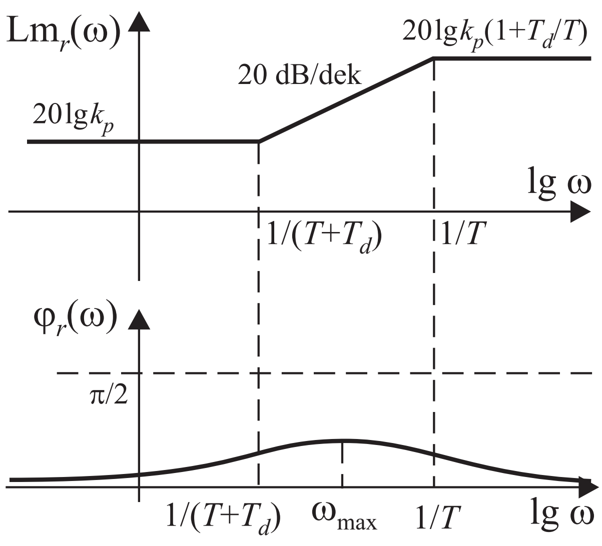
Rys. 87.

$$G_r(j\omega) = k_p\left(1 + \frac{jT_d\omega}{1+jT\omega}\right) = k_p\frac{1+j(T+T_d)\omega}{1+jT\omega}$$

$$\mathrm{Lm}_r(\omega) = 20\lg k_p + 20\lg\sqrt{1+(T+T_d)^2\omega^2} - 20\lg\sqrt{1+T^2\omega^2} \approx$$

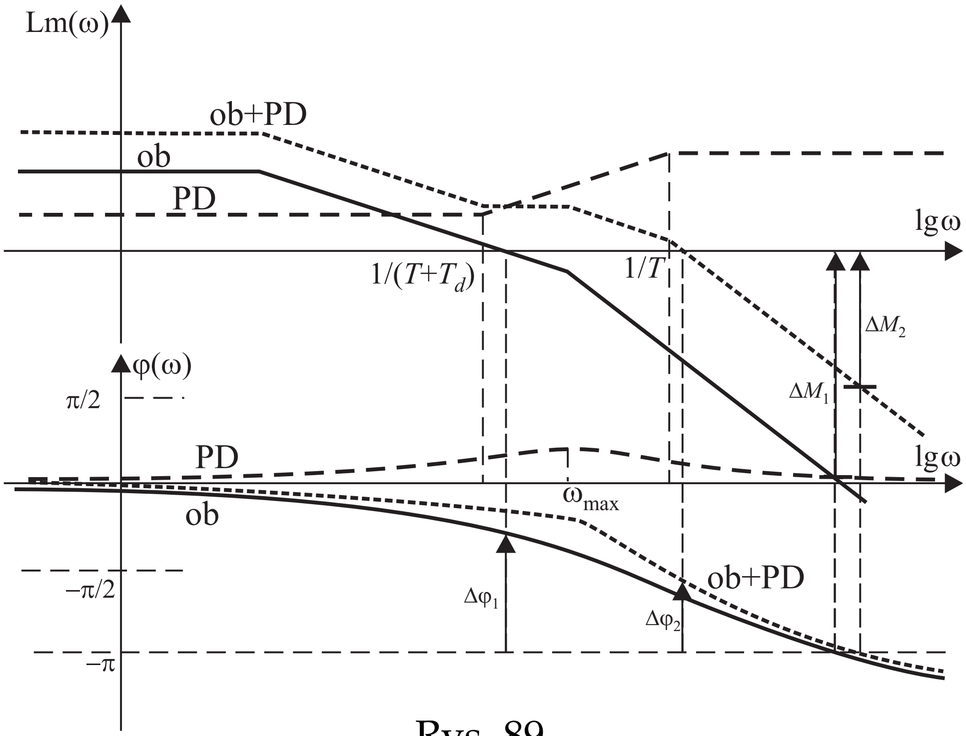
$$\approx \begin{cases} 20\lg k_p, & \omega < 1/(T+T_d) \\ 20\lg k_p + 20\lg\big((T+T_d)\omega\big), & 1/(T+T_d) \leqslant \omega < 1/T \\ \underbrace{20\lg k_p + 20\lg\big((T+T_d)\omega\big) - 20\lg(T\omega)}_{=20\lg\left(k_p(T+T_d)/T\right) = 20\lg\left(k_p(1+T_d/T)\right)}, & \omega \geqslant 1/T \end{cases}$$

$$\varphi_r(\omega) = \mathrm{arctg}\big((T+T_d)\omega\big) - \mathrm{arctg}(T\omega), \qquad \omega_{\max} = \frac{1}{\sqrt{T(T+T_d)}}$$



Rys. 88.

$$\frac{1}{sT_i}(1 + T_d s) = \frac{T_d}{T_i}\left(1 + \frac{1}{T_d s}\right)$$



Rys. 89.

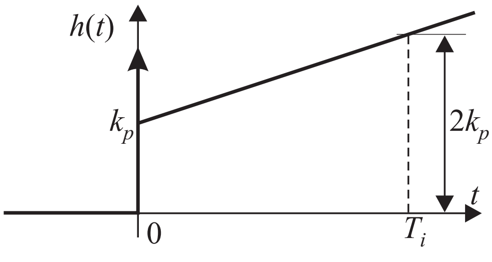
e) regulator typu PID (proporcjonalno-całkująco-różniczkujący)

$*$ regulator typu PID idealny

$$G_r(s) = \frac{U(s)}{E(s)} = k_p\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{90}$$

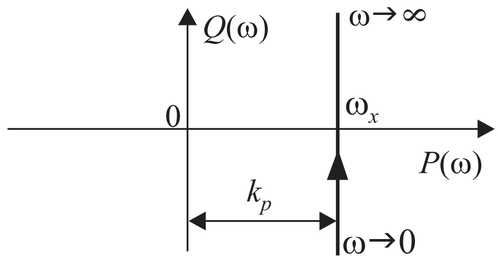
$$u(t) = k_p\left(e(t) + \frac{1}{T_i}\int\limits_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right)$$

$$h(t) = \mathcal{L}^{-1}\left\{G_r(s)\frac{1}{s}\right\} = k_p\left(1 + \frac{t}{T_i} + T_d\delta(t)\right)\mathbb{1}(t) \tag{91}$$



Rys. 90.

$$G_r(j\omega) = k_p\left(1 + \frac{1}{jT_i\omega} + jT_d\omega\right) = k_p\left[1 + j\left(T_d\omega - \frac{1}{T_i\omega}\right)\right]$$

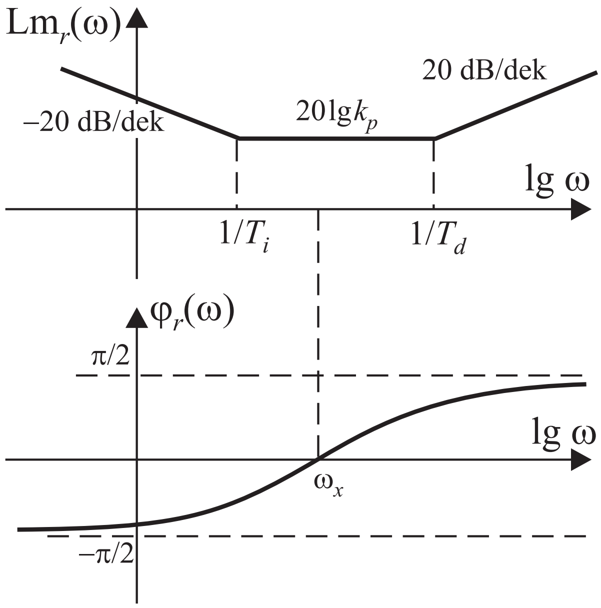


Rys. 91. $\omega_x = 1/\sqrt{T_iT_d}$

$$\mathrm{Lm}_r(\omega) = 20\lg k_p + 20\lg\sqrt{1 + \left(T_d\omega - \frac{1}{T_i\omega}\right)^2} = \{\text{zał. } T_i > T_d\} \approx$$

$$\approx \begin{cases} 20\lg k_p - 20\lg(T_i\omega), & \omega < 1/T_i \\ 20\lg k_p, & 1/T_i \leqslant \omega < 1/T_d \\ 20\lg k_p + 20\lg(T_d\omega), & \omega \geqslant 1/T_d \end{cases}$$

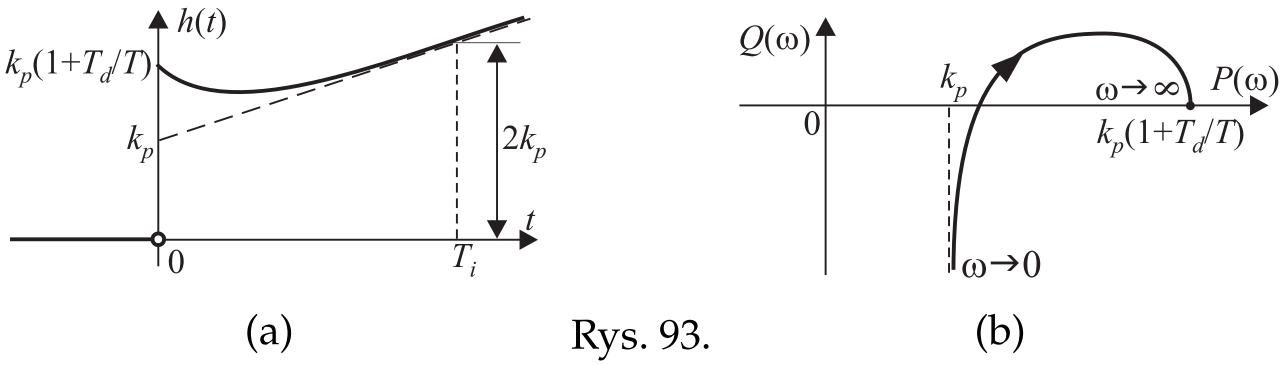
$$\varphi_r(\omega) = \mathrm{arctg}\left(T_d\omega - \frac{1}{T_i\omega}\right)$$



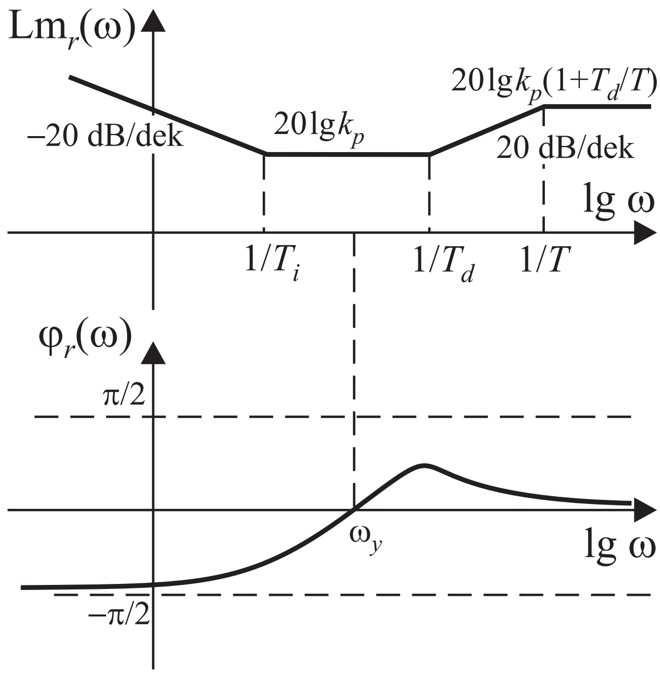
Rys. 92. zał. $T_i > T_d$

∗ regulator typu PID rzeczywisty

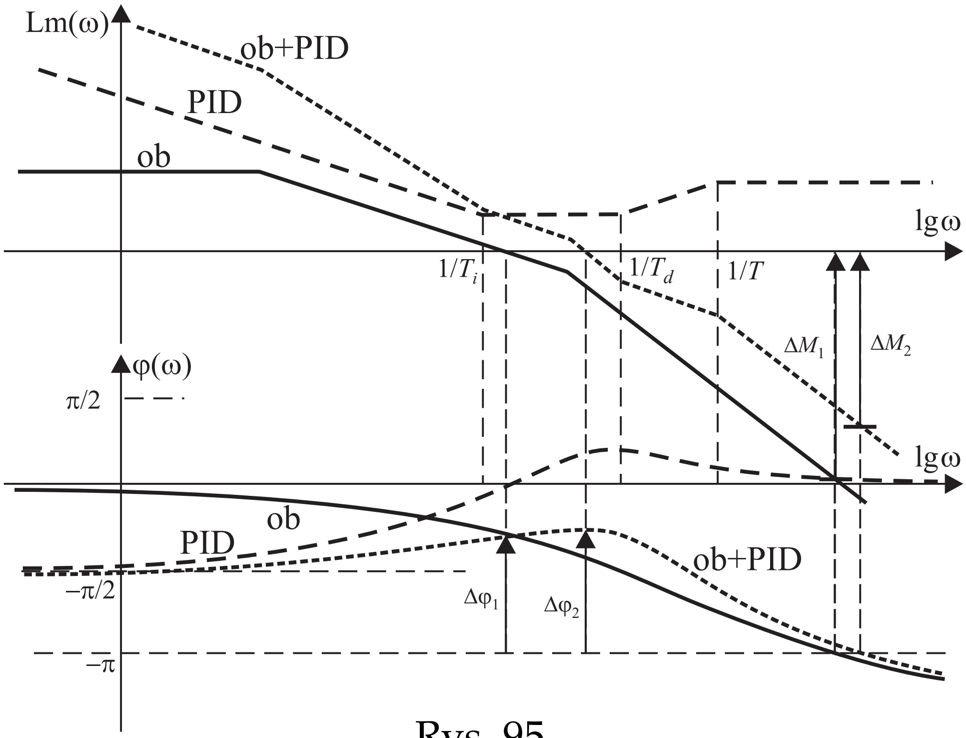
$$G_r(s) = k_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + Ts}\right) \Rightarrow h(t) = k_p\left(1 + \frac{t}{T_i} + \frac{T_d}{T}e^{-t/T}\right)\mathbb{1}(t) \tag{92}$$



Rys. 93.



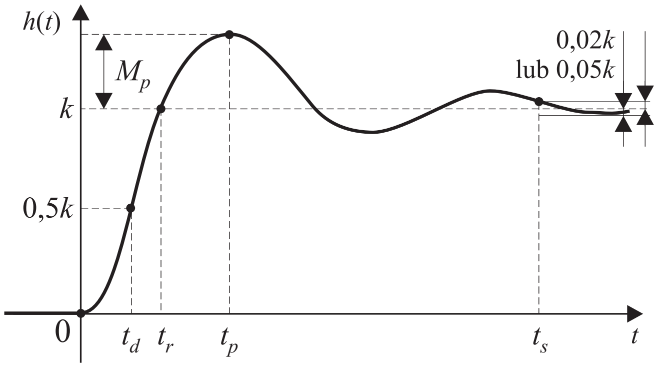
Rys. 94. zał. $T_i > T_d > T$, $\omega_y = \dfrac{1}{\sqrt{T_i T_d - T^2}}$



Rys. 95.

Własności regulatorów ciągłych

odpowiedź układu zamkniętego	czas narastania t_r	przeregulowanie M_p, $\varkappa$	czas ustalania t_s	uchyb ustalony e_u
$k_p \nearrow$	$\searrow$	$\nearrow$	$\approx$ const	$\searrow$
$T_i \nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\to 0$
$T_d \nearrow$	$\approx$ const	$\searrow$	$\searrow$	$\approx$ const



Rys. 96.

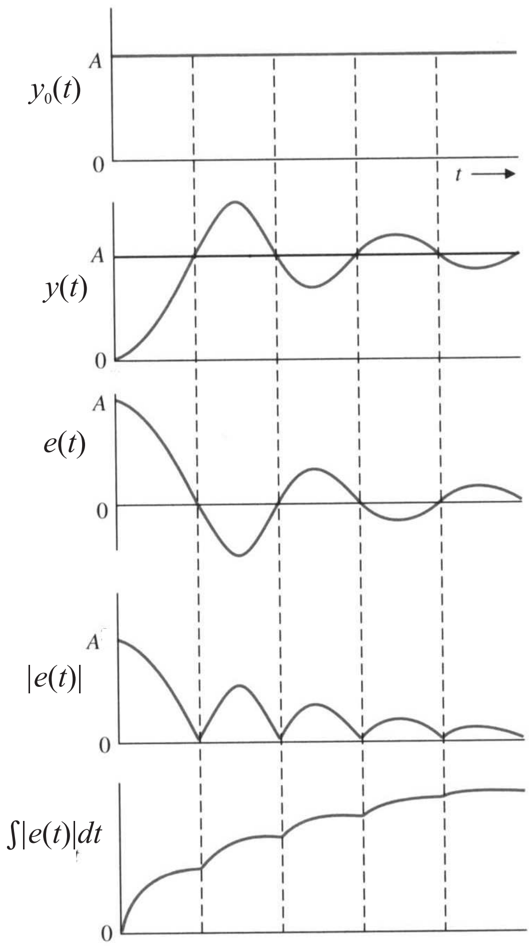
28. Całkowe wskaźniki jakości

<u>Wskaźnik ISE</u> (ang. *integral of the squared error*):

$$I_2 = \int_0^T e^2(t)\,dt \tag{93}$$

<u>Wskaźnik IAE</u> (ang. *integral of the absolute magnitude of the error*):

$$I_a = \int_0^T |e(t)|\,dt \tag{94}$$



Rys. 97.

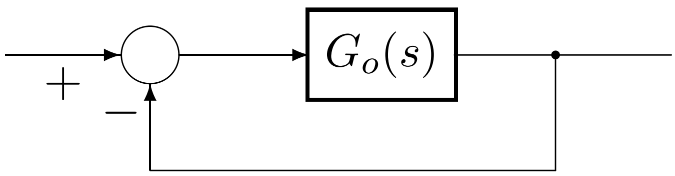
Wskaźnik ITAE (ang. *integral of time multiplied by the absolute magnitude of the error*):

$$I_{ta} = \int_0^T t|e(t)|\,dt \tag{95}$$

Wskaźnik ITSE (ang. *integral of time multiplied by the squared error*):

$$I_{t2} = \int_0^T te^2(t)\,dt \tag{96}$$

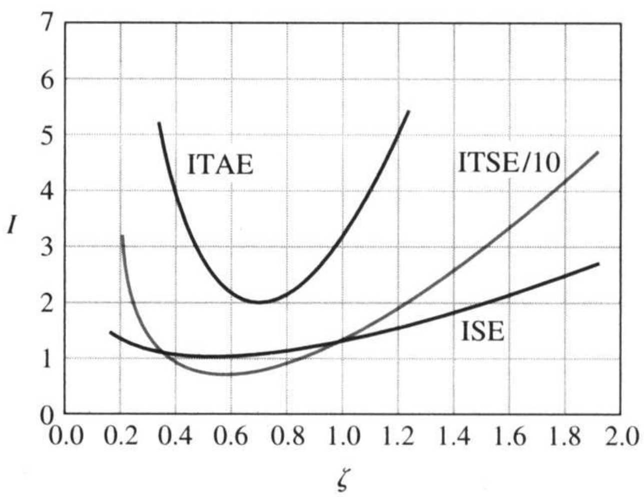
Przykład



Rys. 98.

$$G_o(s) = \frac{1}{s(s+2\zeta)} \quad \Rightarrow \quad G(s) = \frac{G_o(s)}{1+G_o(s)} = \frac{1}{s^2+2\zeta s+1}$$

$$\omega_n = 1, \qquad k = 1$$



Rys. 99.