22. Statyzm i astatyzm ciągłych układów regulacji – c.d.

$$G_o(s) = \frac{L_o(s)}{M_o(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \dots b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0}, \ m \leqslant n \quad \Rightarrow$$

$$\Rightarrow \quad k_o = \lim_{s\to 0} G_o(s) = \lim_{s\to 0} \frac{L_o(s)}{M_o(s)} = \frac{b_0}{a_0}$$

c) układ astatyczny 1-go rzędu ($l = 1$), $y_0(t) = At\mathbb{1}(t)$:

$$e_u = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sG_e(s)Y_0(s) = \lim_{s\to 0} sG_e(s)\frac{A}{s^2} =$$

$$= \lim_{s\to 0} \frac{A}{\left(1 + \frac{L_o(s)}{sM_o(s)}\right) s} = \lim_{s\to 0} \frac{AM_o(s)}{sM_o(s) + L_o(s)} = \frac{A}{k_o}$$

23. Dokładność statyczna

a) odpowiedź skokowa, $y_0(t) = A\mathbb{1}(t)$:

$$e_u = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sG_e(s)Y_0(s) = \lim_{s\to 0} sG_e(s)\frac{A}{s} =$$

$$= \lim_{s\to 0} s\frac{1}{1 + G_o(s)}\frac{A}{s} = \lim_{s\to 0} \frac{A}{1 + G_o(s)} = \frac{A}{1 + k_p} \qquad (71)$$

$$k_p = k_o = \lim_{s\to 0} G_o(s) \quad – \quad \text{stała uchybu położenia} \qquad (72)$$

przy założeniu, że ukł. jest stabilny

$$l = 0 \quad \Rightarrow \quad k_p < \infty \quad \Rightarrow \quad e_u = \frac{A}{1 + k_p}$$

$$l \geqslant 1 \quad \Rightarrow \quad k_p = \lim_{s\to 0} \frac{L_o(s)}{s^l M_o(s)} = \infty \quad \Rightarrow \quad e_u = 0$$

b) odpowiedź na sygnał liniowo narastający $y_0(t) = At\mathbb{1}(t)$:

$$e_u = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sG_e(s)Y_0(s) = \lim_{s\to 0} sG_e(s)\frac{A}{s^2} =$$

$$= \lim_{s\to 0} s\frac{1}{1+G_o(s)}\frac{A}{s^2} = \lim_{s\to 0}\frac{A}{s+sG_o(s)} = \lim_{s\to 0}\frac{A}{sG_o(s)} = \frac{A}{k_v} \quad (73)$$

$$k_v = \lim_{s\to 0} sG_o(s) \quad \text{– stała uchybu prędkościowego} \quad (74)$$

$$l = 0 \quad \Rightarrow \quad k_v = 0 \quad \Rightarrow \quad e_u \to \infty$$

$$l = 1 \quad \Rightarrow \quad k_v = \lim_{s\to 0} s\frac{L_o(s)}{sM_o(s)} < \infty \quad \Rightarrow \quad e_u = \frac{A}{k_v}$$

$$l \geqslant 2 \quad \Rightarrow \quad k_v = \lim_{s\to 0} s\frac{L_o(s)}{s^l M_o(s)} = \infty \quad \Rightarrow \quad e_u = 0$$

c) odpowiedź na sygnał paraboliczny $y_0(t) = \frac{A}{2}t^2\mathbb{1}(t)$:

$$e_u = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sG_e(s)Y_0(s) = \lim_{s\to 0} sG_e(s)\frac{A}{s^3} = \quad (75)$$

$$= \lim_{s\to 0} s\frac{1}{1+G_o(s)}\frac{A}{s^3} = \lim_{s\to 0}\frac{A}{s^2+s^2G_o(s)} = \lim_{s\to 0}\frac{A}{s^2G_o(s)} = \frac{A}{k_a}$$

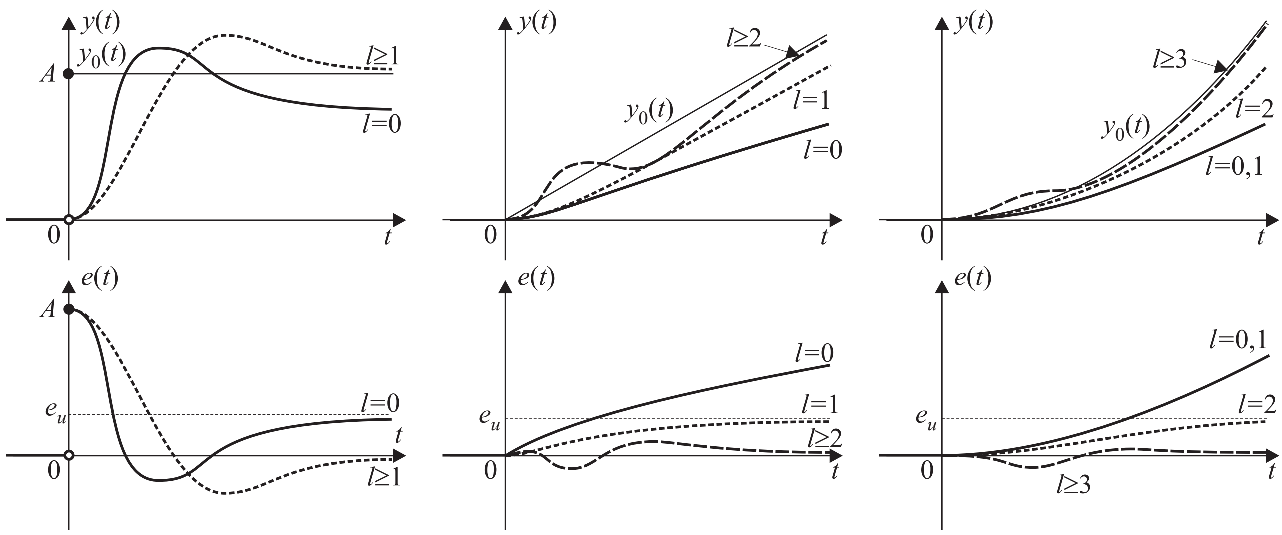
$$k_a = \lim_{s\to 0} s^2G_o(s) \quad \text{– stała uchybu przyspieszeniowego} \quad (76)$$

$$l = 0,1 \quad \Rightarrow \quad k_a = 0 \quad \Rightarrow \quad e_u \to \infty$$

$$l = 2 \quad \Rightarrow \quad k_a = \lim_{s\to 0} s^2\frac{L_o(s)}{s^2M_o(s)} < \infty \quad \Rightarrow \quad e_u = \frac{A}{k_a}$$

$$l \geqslant 3 \quad \Rightarrow \quad k_a = \lim_{s\to 0} s^2\frac{L_o(s)}{s^l M_o(s)} = \infty \quad \Rightarrow \quad e_u = 0$$

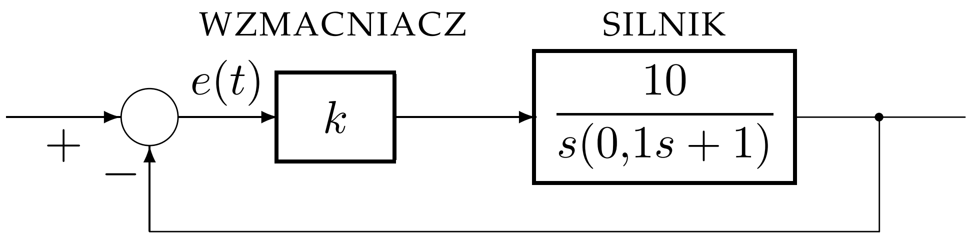
e_u	$l=0$	$l=1$	$l=2$	$l=3$
$y_0(t) = A\mathbb{1}(t)$	$\frac{A}{1+k_p}$	0	0	0
$y_0(t) = At\mathbb{1}(t)$	∞	$\frac{A}{k_v}$	0	0
$y_0(t) = \frac{A}{2}t^2\mathbb{1}(t)$	∞	∞	$\frac{A}{k_a}$	0



(a) (b) (c)

Rys. 68.

Przykład

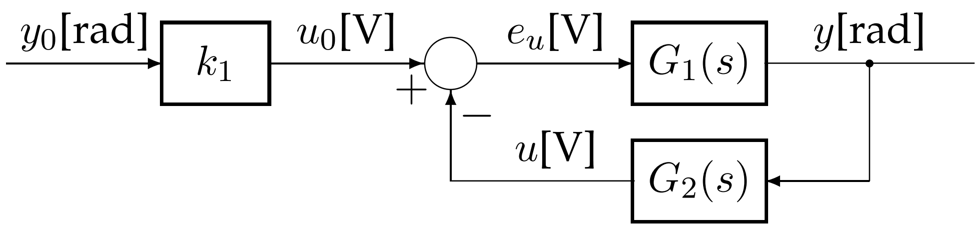


Rys. 69.

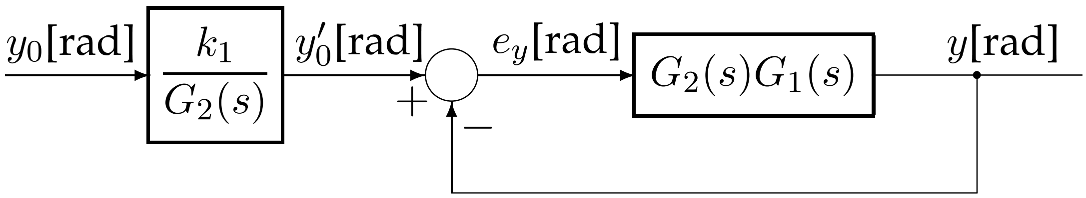
$$k_p = \lim_{s\to 0} kG(s) = \infty, \quad k_v = \lim_{s\to 0} skG(s) = 10k, \quad k_a = \lim_{s\to 0} s^2 kG(s) = 0$$

$$e_{up} = 0, \quad e_{uv} = \frac{A}{10k}, \quad e_{ua} \to \infty$$

24. Uchyb ustalony w układach z niejednostkowym (elastycznym) sprzężeniem zwrotnym

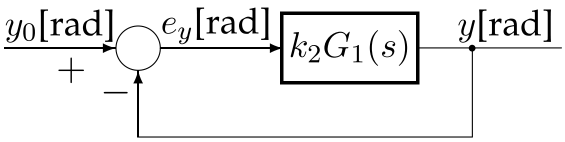


Rys. 70.



Rys. 71.

Zazwyczaj $G_2(s) = k_2$ lub $\lim_{s\to 0} G_2(s) = k_2$. Możemy wtedy dobrać k_1 tak, by $k_1 = k_2$, a wtedy:



Rys. 72.

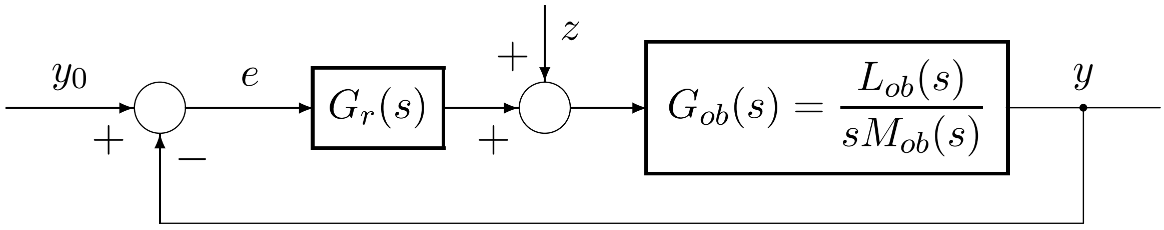
Przykład

$$G_1(s) = \frac{40}{s+5} = \frac{8}{0{,}2s+1}, \quad G_2(s) = \frac{20}{s+10} = \frac{2}{0{,}1s+1} \Rightarrow k_2 = 2$$

dobieramy $k_1 = k_2 = 2$ i wtedy $e_u = \dfrac{1}{1+k_p} = \dfrac{1}{1+2\cdot 8} = \dfrac{1}{17}$.

25. Astatyzm względem sygnału zadanego i zakłócenia

a) obiekt jest astatyczny



Rys. 73.

$$G_e(s) = \left.\frac{E(s)}{Y_0(s)}\right|_{z(t)\equiv 0} = \frac{1}{1 + G_r(s)G_{ob}(s)} = \frac{1}{1 + G_r(s)\frac{L_{ob}(s)}{sM_{ob}(s)}} =$$
$$= \frac{sM_{ob}(s)}{sM_{ob}(s) + G_r(s)L_{ob}(s)} \tag{77}$$

$$e_u\Big|_{z(t)\equiv 0} = \lim_{s\to 0} sG_e(s)Y_0(s) = \lim_{s\to 0} sG_e(s)\frac{1}{s} =$$
$$= \lim_{s\to 0} \frac{sM_{ob}(s)}{sM_{ob}(s) + G_r(s)L_{ob}(s)} = 0 \tag{78}$$

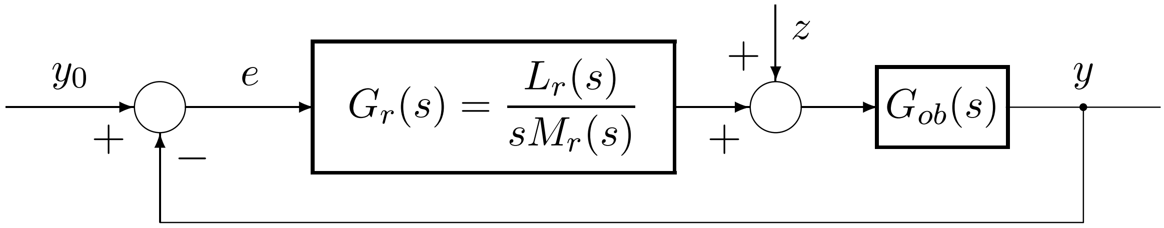
Układ jest astatyczny względem wartości zadanej.

$$G_z(s) = \left.\frac{Y(s)}{Z(s)}\right|_{y_0(t)\equiv 0} = \left.\frac{-E(s)}{Z(s)}\right|_{y_0(t)\equiv 0} = \frac{G_{ob}(s)}{1 + G_r(s)G_{ob}(s)} =$$
$$= \frac{\frac{L_{ob}(s)}{sM_{ob}(s)}}{1 + G_r(s)\frac{L_{ob}(s)}{sM_{ob}(s)}} = \frac{L_{ob}(s)}{sM_{ob}(s) + G_r(s)L_{ob}(s)} \tag{79}$$

$$e_u\Big|_{y_0(t)\equiv 0} = \lim_{s\to 0} sE(s)\Big|_{y_0(t)\equiv 0} = -\lim_{s\to 0} sG_z(s)Z(s) = -\lim_{s\to 0} sG_z(s)\frac{1}{s} =$$
$$= \lim_{s\to 0} \frac{-L_{ob}(s)}{sM_{ob}(s) + G_r(s)L_{ob}(s)} \neq 0 \tag{80}$$

Układ jest statyczny względem zakłócenia.

b) regulator jest astatyczny



Rys. 74.

$$G_e(s) = \left.\frac{E(s)}{Y_0(s)}\right|_{z(t)\equiv 0} = \frac{1}{1 + G_r(s)G_{ob}(s)} = \frac{1}{1 + \frac{L_r(s)}{sM_r(s)}G_{ob}(s)} =$$
$$= \frac{sM_r(s)}{sM_r(s) + L_r(s)G_{ob}(s)} \tag{81}$$

$$e_u\Big|_{z(t)\equiv 0} = \lim_{s\to 0} sG_e(s)Y_0(s) = \lim_{s\to 0} sG_e(s)\frac{1}{s} =$$
$$= \lim_{s\to 0} \frac{sM_r(s)}{sM_r(s) + L_r(s)G_{ob}(s)} = 0 \tag{82}$$

Układ jest astatyczny względem wartości zadanej.

$$G_z(s) = \left.\frac{Y(s)}{Z(s)}\right|_{y_0(t)\equiv 0} = \left.\frac{-E(s)}{Z(s)}\right|_{y_0(t)\equiv 0} = \frac{G_{ob}(s)}{1 + G_r(s)G_{ob}(s)} =$$
$$= \frac{G_{ob}(s)}{1 + \frac{L_r(s)}{sM_r(s)}G_{ob}(s)} = \frac{sM_r(s)G_{ob}(s)}{sM_r(s) + L_r(s)G_{ob}(s)} \tag{83}$$

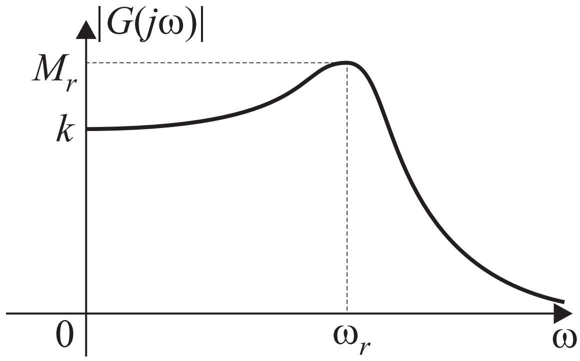
$$e_u\Big|_{y_0(t)\equiv 0} = \lim_{s\to 0} sG_z(s)Z(s) = -\lim_{s\to 0} sG_z(s)\frac{1}{s} =$$
$$= \lim_{s\to 0} \frac{-sM_r(s)G_{ob}(s)}{sM_r(s) + L_r(s)G_{ob}(s)} = 0 \tag{84}$$

Układ jest również astatyczny względem zakłócenia.

26. Wskaźniki częstotliwościowe jakości regulacji

a) zapas modułu λ i ΔM oraz zapas fazy $\Delta\varphi$,

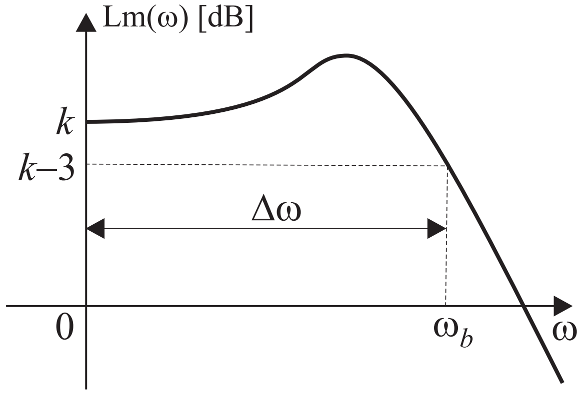
b) moduł rezonansowy M_r i pulsacja rezonansowa ω_r (dla układu zamkniętego)



Rys. 75.

$$M_r = \frac{k}{2\zeta\sqrt{1-\zeta^2}}, \qquad \omega_r = \omega_n\sqrt{1-2\zeta^2}, \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

c) pulsacja odcięcia ω_b i szerokość pasma przepuszczania $\Delta\omega$



Rys. 76.

$$\mathrm{Lm}(0) - \mathrm{Lm}(\omega_b) = +3[\mathrm{dB}]$$

$$0 \leqslant \omega \leqslant \omega_b$$