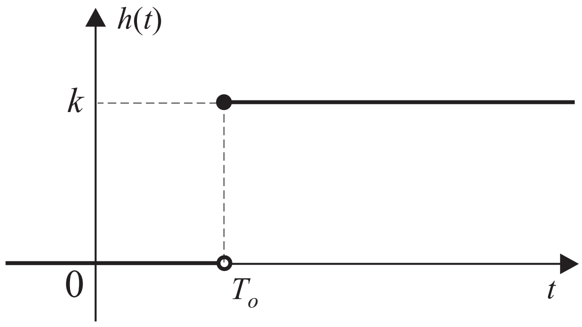
18. Element opóźniający

$$G(s) = \frac{Y(s)}{U(s)} = ke^{-sT_o}, \quad k > 0 \quad \rightarrow \tag{57}$$

$$\rightarrow \quad y(t) = \mathcal{L}^{-1}\left\{G(s)U(s)\right\} = k\,u(t-T_o)$$

$$h(t) = \mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\} = k\,\mathbb{1}(t-T_o), \quad g(t) = \mathcal{L}^{-1}\left\{G(s)\right\} = k\,\delta(t-T_o)$$

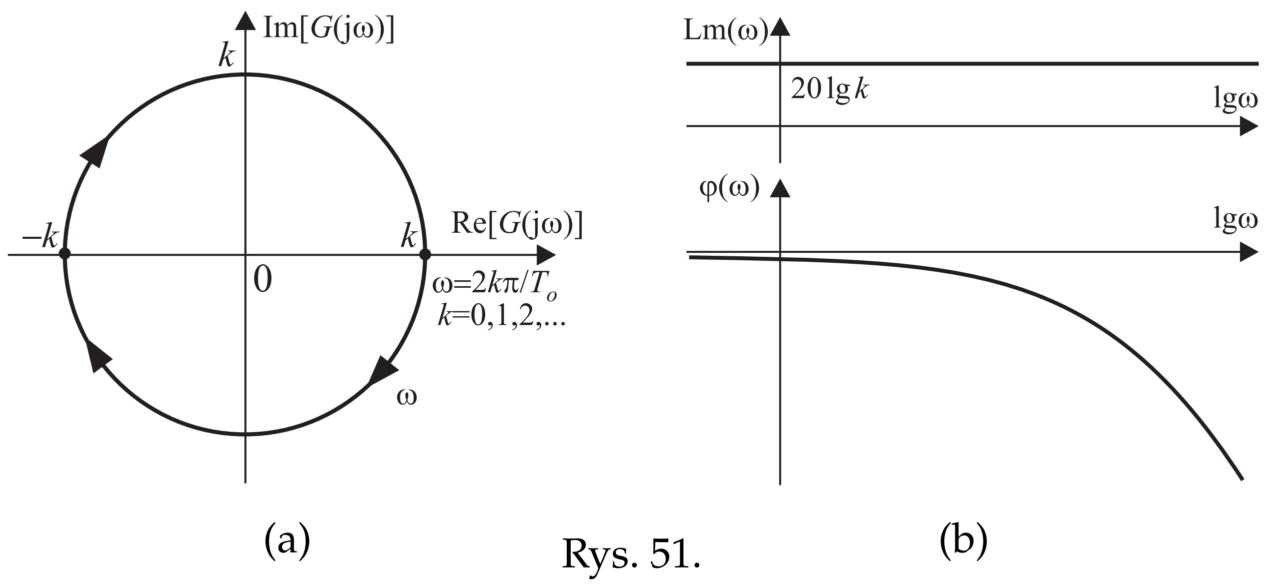
$$G(j\omega) = G(s)|_{s=j\omega} = ke^{-j\omega T_o} = k\big(\cos(\omega T_o) - j\sin(\omega T_o)\big) \tag{58}$$



Rys. 50.

$$\mathrm{Lm}(\omega) = 20\lg|G(j\omega)| = 20\lg k + \underbrace{20\lg|e^{-j\omega T_o}|}_{=0} = 20\lg k,$$

$$\varphi(\omega) = \arg\big(G(j\omega)\big) = -\omega T_o \tag{59}$$

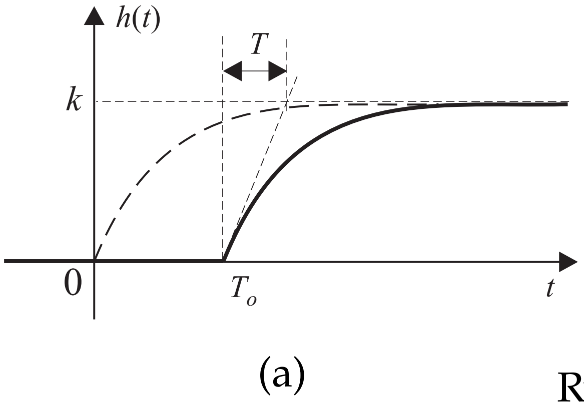


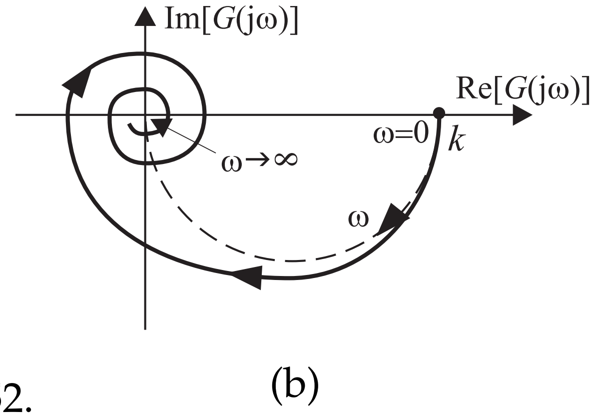
Rys. 51.

Przykład (element inercyjny 1-go rzędu z opóźnieniem)

$$G(s) = \frac{k}{1+sT}e^{-sT_o},\ k>0 \ \rightarrow\ h(t) = \mathcal{L}^{-1}\left\{\frac{G(s)}{s}\right\} = k\left(1 - e^{-\frac{t-T_o}{T}}\right)\mathbb{1}(t-T_o)$$

$$|G(j\omega)| = \frac{k}{\sqrt{1+\omega^2T^2}}\cdot 1, \qquad \varphi(\omega) = -\mathrm{arctg}(\omega T) - \omega T_o$$



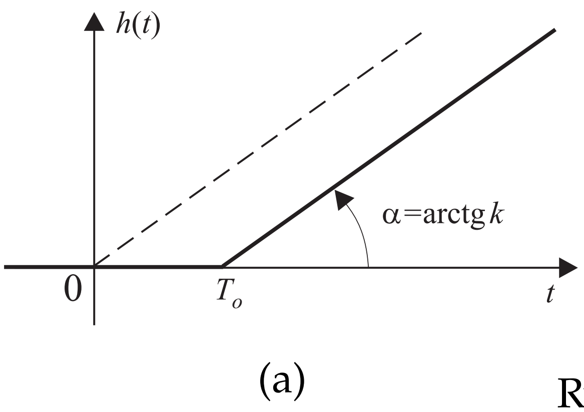


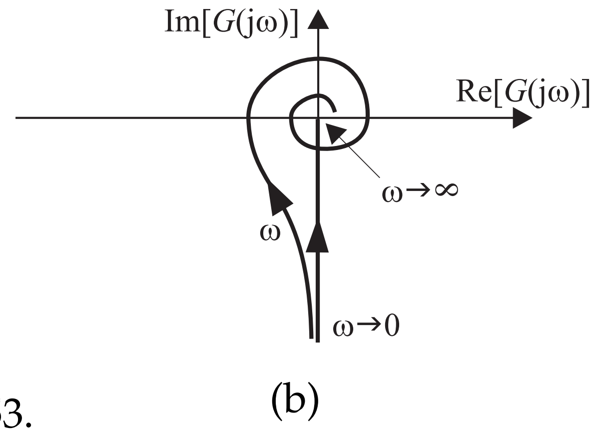
Rys. 52.

Przykład (element całkujący idealny z opóźnieniem)

$$G(s) = \frac{k}{s}e^{-sT_o},\ \ k>0 \quad \rightarrow \quad h(t) = k(t-T_o)\mathbb{1}(t-T_o)$$

$$|G(j\omega)| = \frac{k}{\omega}\cdot 1, \qquad \varphi(\omega) = -\frac{\pi}{2} - \omega T_o$$





Rys. 53.

19. Przybliżanie transmitancji operatorowej obiektem z opóźnieniem

a) elementu inercyjnego n-tego rzędu ($n > 1$) transmitancją elementu inercyjnego 1-go rzędu z opóźnieniem

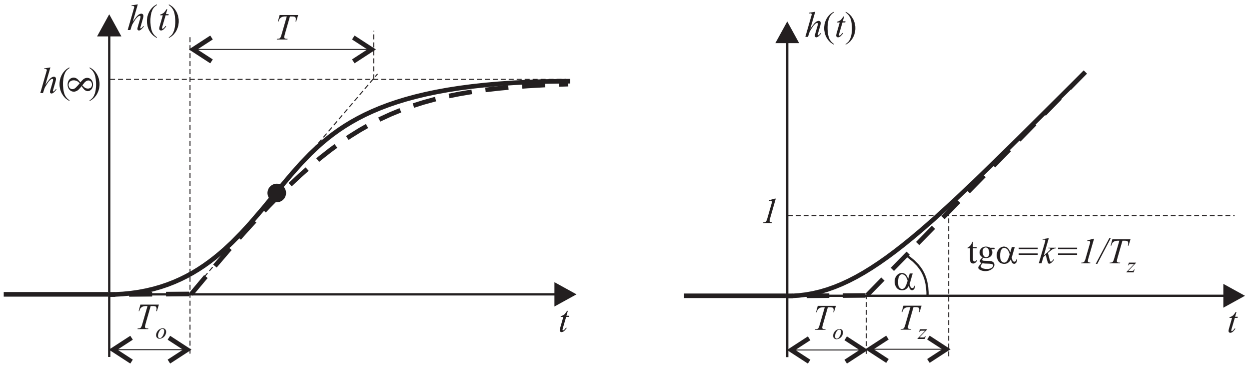
$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{(1+sT_1)(1+sT_2)\dots(1+sT_n)}$$

$$u(t) = A\mathbb{1}(t) \quad \Rightarrow \quad k = \frac{h(\infty)}{A} = \frac{y(\infty)}{A}$$

$$T_1 = ?, \quad T_2 = ?, \quad \dots \quad T_n = ?$$

Np.

$$G(s) = \frac{k}{(1+sT_1)(1+sT_2)} \Rightarrow h(t) = k\left(1 - \frac{T_1 e^{-t/T_1} - T_2 e^{-t/T_2}}{T_1 - T_2}\right)\mathbb{1}(t)$$



(a) Rys. 54. (b)

$$G(s) \approx G_a(s) = \frac{k}{1+sT}e^{-sT_o}, \qquad k = \frac{h(\infty)}{A} \tag{60}$$

$$h_a(t) = k\left(1 - e^{-\frac{t-T_o}{T}}\right)\mathbb{1}(t-T_o) \tag{61}$$

b) elementu całkującego z inercją n-tego rzędu ($n > 1$)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{k}{s(1+sT_1)(1+sT_2)\dots(1+sT_n)}$$

$$G(s) \approx G_b(s) = \frac{k}{s}e^{-sT_o} \tag{62}$$

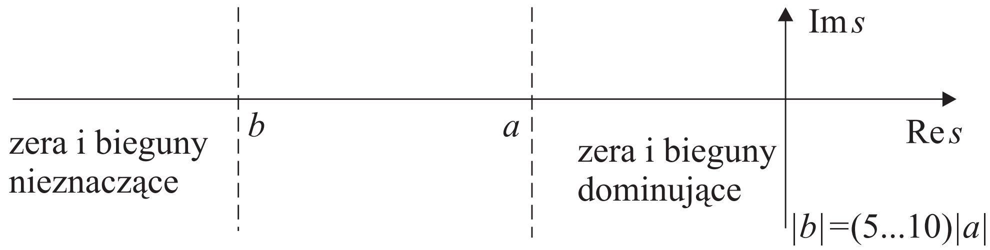
$$h_b(t) = k(t-T_o)\mathbb{1}(t-T_o)$$

20. Redukcja rzędu modelu układu dynamicznego

$$G(s) = \frac{k}{(1+sT_1)(1+sT_2)} \to h(t) = k\left(1 - \frac{T_1 e^{-t/T_1} - T_2 e^{-t/T_2}}{T_1 - T_2}\right)\mathbb{1}(t)$$

$s_1 = -1/T_1, s_2 = -1/T_2,$ zał. $T_2 \ll T_1 \to s_2 \ll s_1$

$$h(0) = 0, h(\infty) = k \to h_{red}(t) = k(1 - e^{-t/T_1})\mathbb{1}(t) \to G_{red} = \frac{k}{1+sT_1}$$

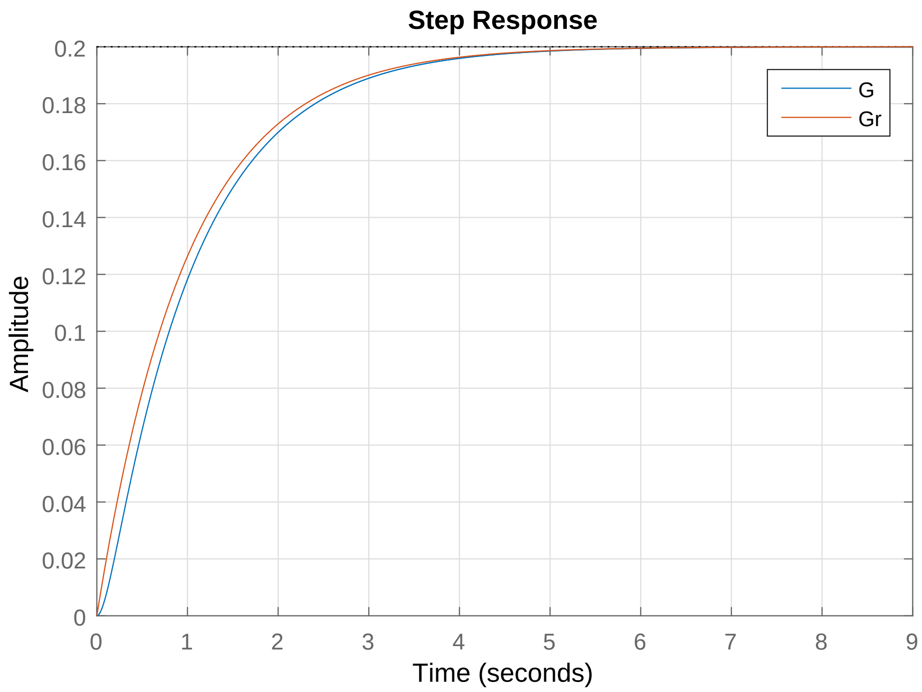


Rys. 55.

Przykład (element inercyjny 2-go rzędu)

$$G(s) = \frac{2}{(s+1)(s+10)} = \frac{2}{10(s+1)(0,1s+1)}$$

$$G_r(s) = \frac{2}{10(s+1)}$$

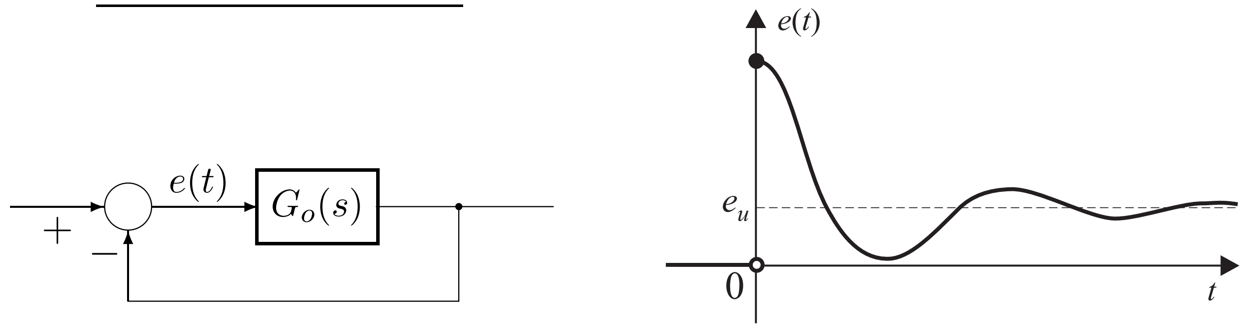


Rys. 56.

21. Wskaźniki czasowe jakości regulacji

a) uchyb ustalony e_u
b) czas opóźnienia t_d
c) czas narastania t_r
d) czas osiągnięcia wartości maksymalnej t_p
e) przeregulowanie M_p lub względne przeregulowanie $\varkappa$
f) czas ustalania (regulacji) t_s

Ad. a) uchyb ustalony e_u

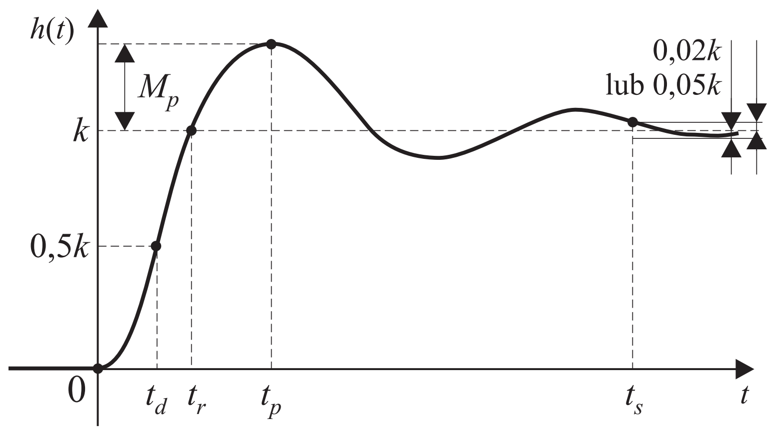


(a) Rys. 57. (b)

$$e(t) = e_u + e_p(t) \quad (63)$$

$$\lim_{t\to\infty} e_p(t) = 0 \quad \Rightarrow \quad \lim_{t\to\infty} e(t) = e_u = \lim_{s\to 0} sE(s) \quad (64)$$

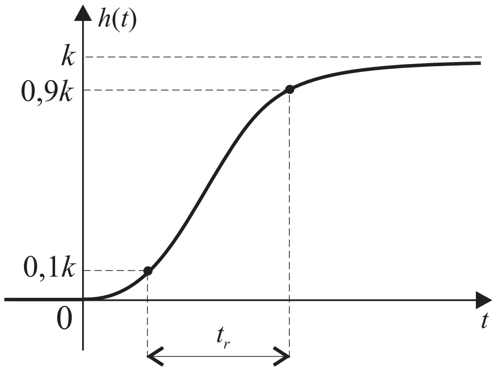
Ad. b) czas opóźnienia t_d

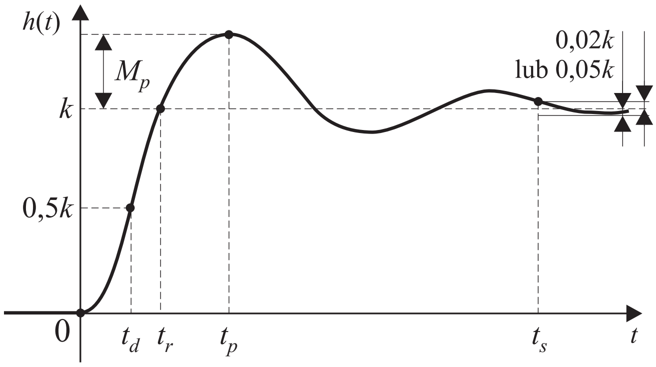


Rys. 58.

Ad. c) czas narastania t_r

$$10\% \to 90\% \quad \text{wartości ustalonej dla} \quad \zeta \geqslant 1$$
$$0\% \to 100\% \quad \text{wartości ustalonej dla} \quad 0 < \zeta < 1$$





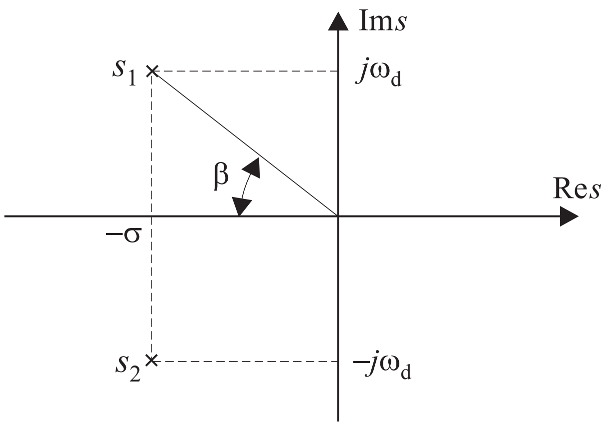
(a) Rys. 59. (b)

$$h(t_r) = k = k\left[1 - e^{-\zeta\omega_n t_r}\left(\cos(\omega_d t_r) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t_r)\right)\right]$$

$$e^{-\zeta\omega_n t_r} \neq 0 \quad \Rightarrow \quad \cos(\omega_d t_r) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t_r) = 0$$

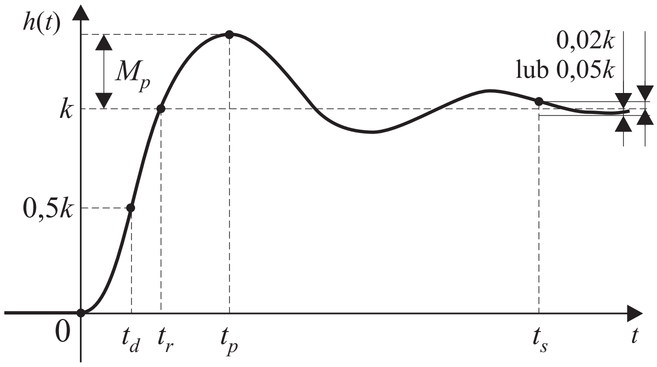
$$\text{tg}(\omega_d t_r) = -\frac{\sqrt{1-\zeta^2}}{\zeta} = -\frac{\omega_d}{\sigma}, \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}, \quad \sigma = \omega_n\zeta$$

$$\boxed{t_r = \frac{1}{\omega_d}\text{arctg}\frac{\omega_d}{-\sigma} = \frac{\pi - \beta}{\omega_d}} \tag{65}$$



Rys. 60. $s_1 = -\sigma + j\omega_d$, $s_2 = -\sigma - j\omega_d$

Ad. d) czas osiągnięcia wartości maksymalnej t_p



Rys. 61.

$$h(t)=k\left[1-e^{-\zeta\omega_n t}\left(\cos(\omega_d t)+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right)\right] \quad \text{dla} \quad t \geqslant 0$$

$$\frac{dh(t)}{dt}=k\zeta\omega_n e^{-\zeta\omega_n t}\left(\cos(\omega_d t)+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t)\right)+$$

$$+\,ke^{-\zeta\omega_n t}\left(\omega_d\sin(\omega_d t)-\frac{\zeta\omega_d}{\sqrt{1-\zeta^2}}\cos(\omega_d t)\right)=$$

$$=ke^{-\zeta\omega_n t}\left[\zeta\omega_n\cos(\omega_d t)+\frac{\zeta^2\omega_n}{\sqrt{1-\zeta^2}}\sin(\omega_d t)+\omega_n\sqrt{1-\zeta^2}\sin(\omega_d t)-\right.$$

$$\left.-\zeta\omega_n\cos(\omega_d t)\right]=k\omega_n\sin(\omega_d t)e^{-\zeta\omega_n t}\left(\frac{\zeta^2}{\sqrt{1-\zeta^2}}+\frac{1-\zeta^2}{\sqrt{1-\zeta^2}}\right)$$

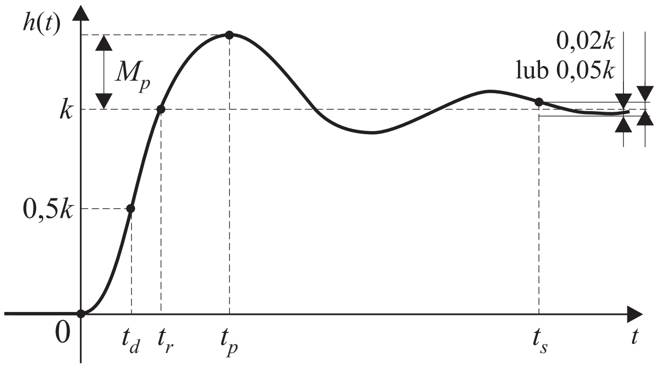
$$\left.\frac{dh(t)}{dt}\right|_{t=t_p}=k\sin(\omega_d t_p)\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t_p}=0 \qquad (66)$$

czyli $\sin(\omega_d t_p)=0$ lub $\omega_d t_p=0,\pi,2\pi,\ldots$

czas t_p odpowiada pierwszemu przeregulowaniu $\omega_d t_p=\pi$:

$$\boxed{t_p=\frac{\pi}{\omega_d}=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}} \qquad (67)$$

Ad. e) przeregulowanie M_p lub względne przeregulowanie $\varkappa$



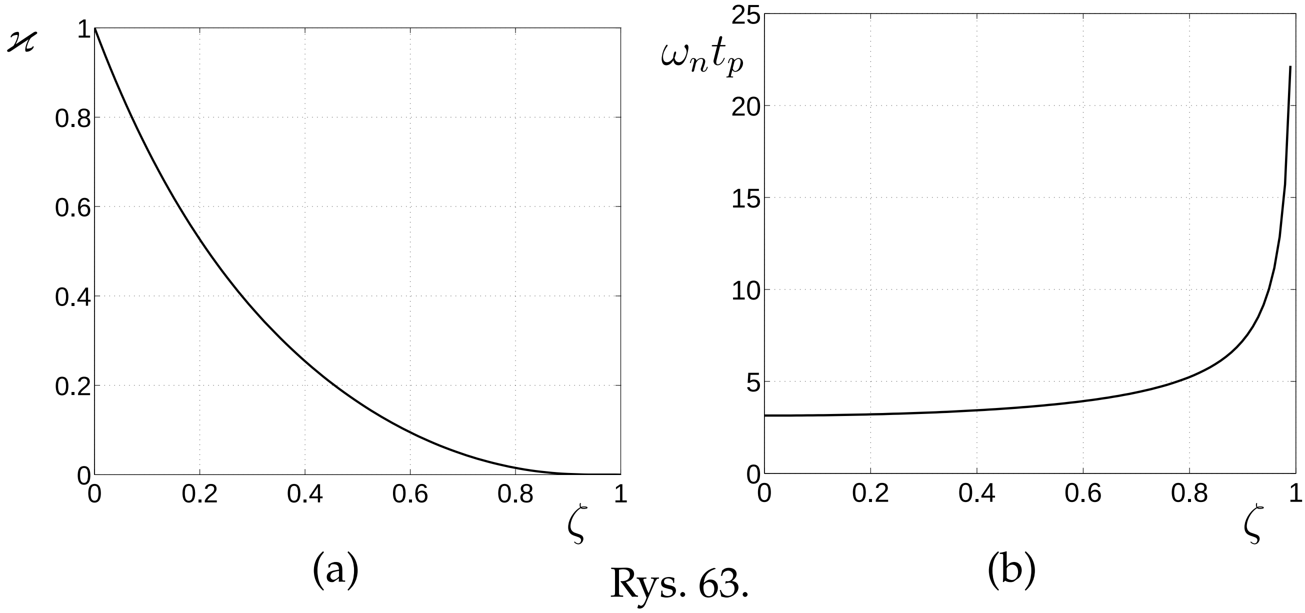
Rys. 62.

$$M_p = h(t_p) - h(\infty) = k\left[1 - e^{-\zeta\omega_n t_p}\left(\cos\pi + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\pi\right)\right] - k =$$

$$= -ke^{-\zeta\omega_n(\pi/\omega_d)}\left(\underbrace{\cos\pi}_{=-1} + \frac{\zeta}{\sqrt{1-\zeta^2}}\underbrace{\sin\pi}_{=0}\right) = ke^{-\frac{\sigma}{\omega_d}\pi} = ke^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}$$

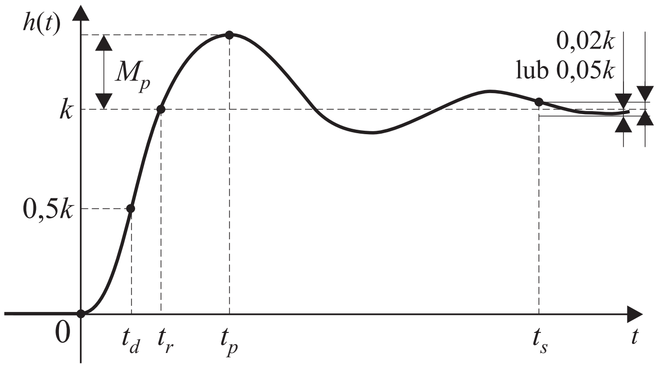
$$\boxed{M_p = ke^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}} \tag{68}$$

$$\boxed{\varkappa = \frac{M_p}{h(\infty)} = \frac{h(t_p) - h(\infty)}{h(\infty)} = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}} \tag{69}$$



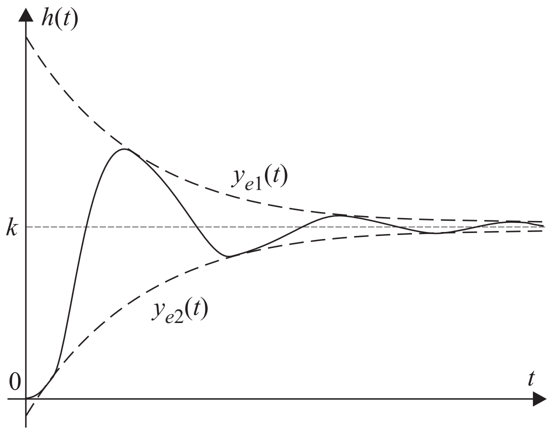
Rys. 63.

Ad. f) czas ustalania (regulacji) t_s



Rys. 64.

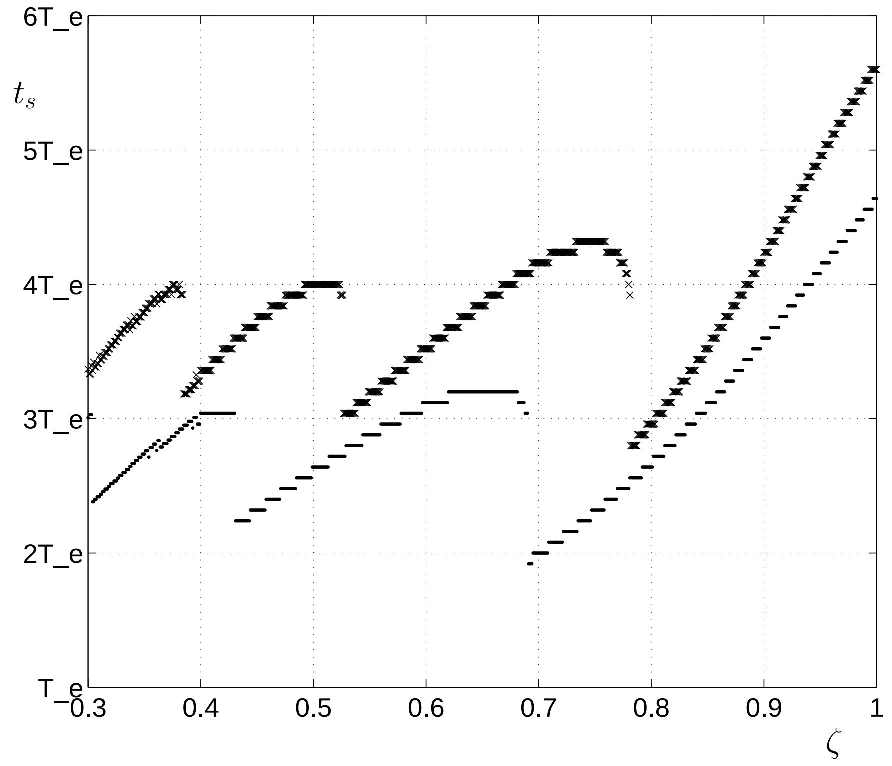
$$y_{e1/2}(t) = k\left(1 \pm \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\right)\mathbb{1}(t), \qquad T_e = \frac{1}{\zeta\omega_n}$$



Rys. 65.

Przyjmuje się, że $t_s = 4T_e = \dfrac{4}{\zeta\omega_n}$ dla kryterium $\pm 2\%$,

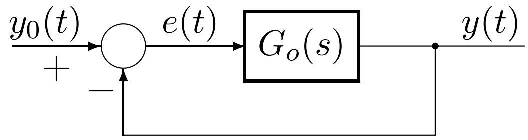
$t_s = 3T_e = \dfrac{3}{\zeta\omega_n}$ dla kryterium $\pm 5\%$



Rys. 66.

22. Statyzm i astatyzm ciągłych układów regulacji

$$G_o(s) = \frac{L_o(s)}{s^l M_o(s)} \tag{70}$$



Rys. 67.

a) układ statyczny, $y_0(t) = A\mathbb{1}(t)$ czyli $Y_0(s) = A/s$:

$$e_u = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sG_e(s)\frac{A}{s} = \lim_{s\to 0} \frac{A}{1+G_o(s)} = \frac{A}{1+k_o} \neq 0$$

b) układ astatyczny 1-go rzędu ($l = 1$), $y_0(t) = A\mathbb{1}(t)$:

$$e_u = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sG_e(s)\frac{A}{s} = \lim_{s\to 0} \frac{A}{1+\frac{L_o(s)}{sM_o(s)}} = \lim_{s\to 0} \frac{AsM_o(s)}{sM_o(s)+L_o(s)} = 0$$