3. Charakterystyki logarytmiczne (wykresy Bodego) – c.d.

$$\text{oktawa} \rightarrow \omega_2 = 2\omega_1 \rightarrow \frac{\lg\omega_b - \lg\omega_a}{\lg 2} = 3{,}32\lg\frac{\omega_b}{\omega_a} \tag{13}$$

$$\text{dekada} \rightarrow \omega_2 = 10\omega_1 \rightarrow \frac{\lg\omega_b - \lg\omega_a}{\lg 10} = \lg\frac{\omega_b}{\omega_a} \tag{14}$$

Przykład (element inercyjny 1-go rzędu)

$$G(j\omega) = \frac{k}{1+j\omega T}, \qquad k > 0$$

$$|G(j\omega)| = \frac{k}{\sqrt{1+\omega^2T^2}}, \qquad \varphi(\omega) = -\text{arctg}(\omega T)$$

$$\text{Lm}(\omega) = 20\lg\frac{k}{\sqrt{1+\omega^2T^2}} = 20\lg k - 20\lg\sqrt{1+\omega^2T^2}$$

$$1+\omega^2T^2 = \begin{cases} 1 & \text{dla } \omega \ll 1/T \\ \omega^2T^2 & \text{dla } \omega \gg 1/T \end{cases}$$

$$1+\omega^2T^2 \approx \begin{cases} 1 & \text{dla } \omega < 1/T \\ \omega^2T^2 & \text{dla } \omega \geqslant 1/T \end{cases}$$

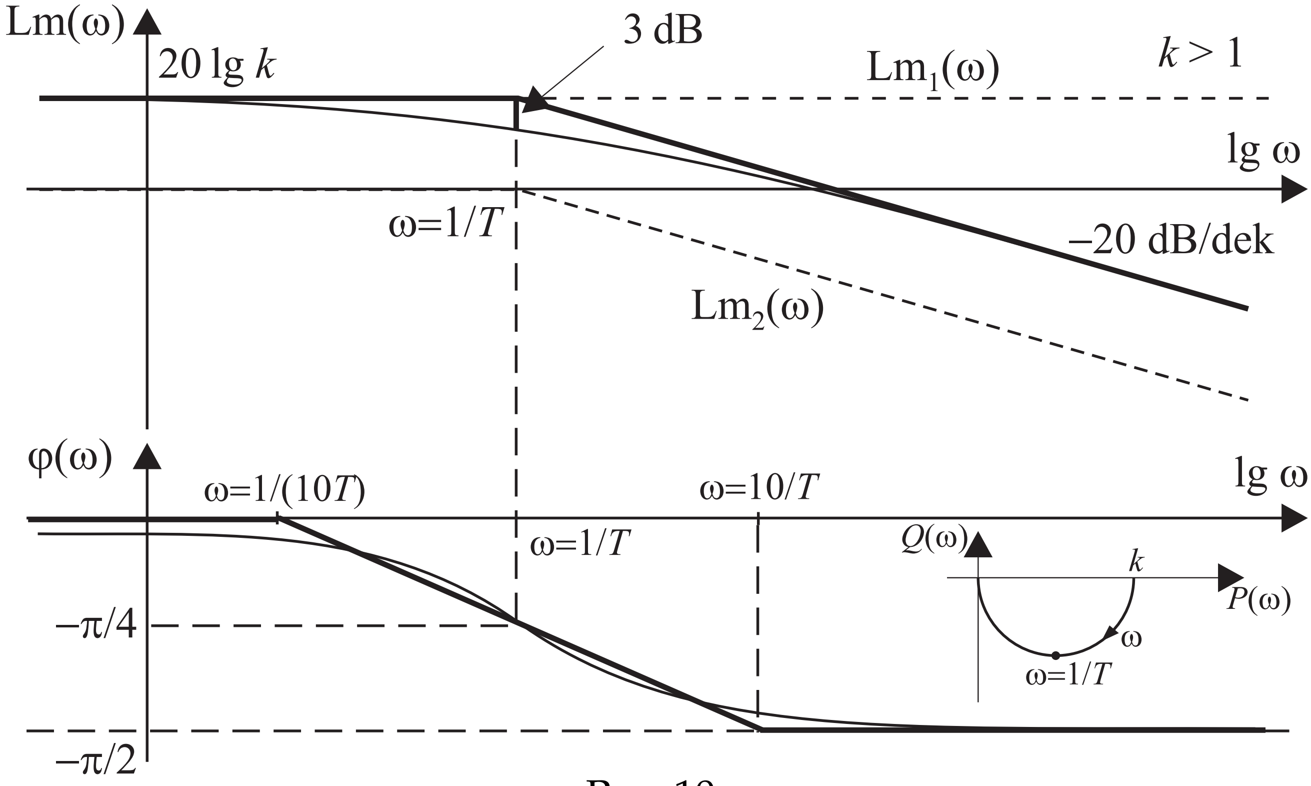
$$\text{Lm}(\omega) \approx \begin{cases} 20\lg k - 20\lg 1 & \text{dla} \quad \omega < 1/T \\ 20\lg k - 20\lg\sqrt{\omega^2T^2} & \text{dla} \quad \omega \geqslant 1/T \end{cases} =$$

$$= \begin{cases} 20\lg k & \text{dla} \quad \omega < 1/T \\ 20\lg k - 20\lg(\omega T) & \text{dla} \quad \omega \geqslant 1/T \end{cases}$$

$$\mathrm{Lm}(\omega) = \mathrm{Lm}_1(\omega) + \mathrm{Lm}_2(\omega), \qquad \mathrm{Lm}_1(\omega) = 20\lg k$$

$$\mathrm{Lm}_2(\omega) = \begin{cases} 0 & \text{dla} \quad \omega < 1/T \\ -20\lg(\omega T) & \text{dla} \quad \omega \geqslant 1/T \end{cases}$$

$$-20\lg(\omega T) = -20\lg\omega + 20\lg(1/T) = 0 \quad \text{dla} \quad \omega = 1/T$$



Rys. 10.

$$\mathrm{Lm}(10\omega_x) - \mathrm{Lm}(\omega_x) = 20\lg k - 20\lg(10\omega_x T) - \big(20\lg k -$$

$$- 20\lg(\omega_x T)\big) = 20\lg\frac{\omega_x T}{10\omega_x T} = -20\left[\frac{\mathrm{dB}}{\mathrm{dek}}\right] = -6\left[\frac{\mathrm{dB}}{\mathrm{okt}}\right]$$

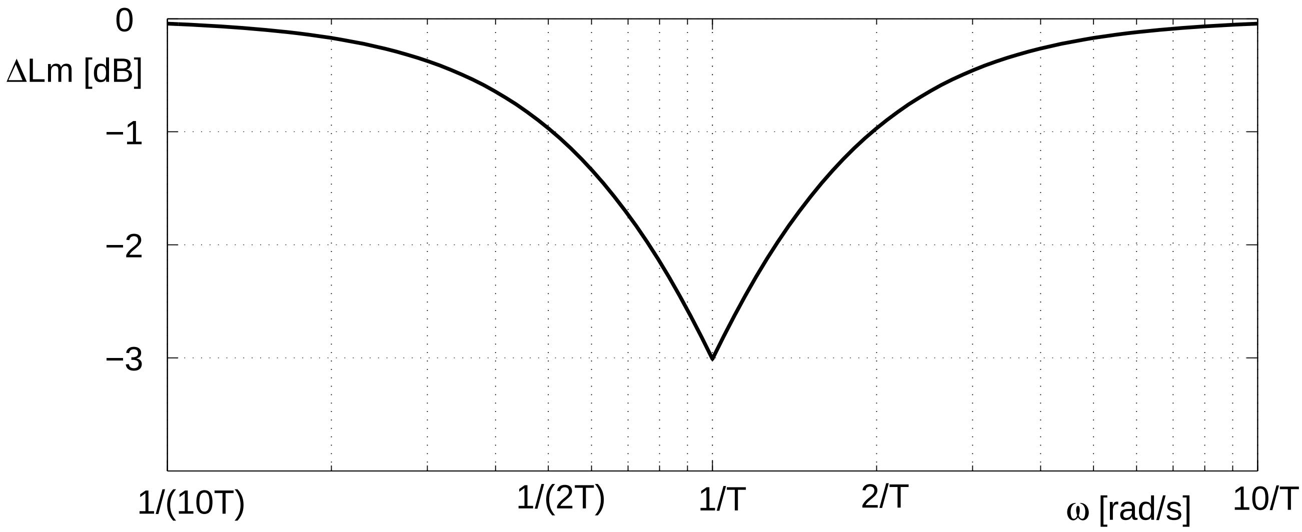
$$\varphi(\omega) = -\mathrm{arctg}(\omega T) \quad \text{dla} \quad k > 0$$

$$\varphi(1/T) = -\pi/4, \qquad \varphi(0) = 0, \qquad \varphi(\infty) \to -\pi/2$$

$$\Delta\mathrm{Lm}(\omega) = \mathrm{Lm}_{dokl}(\omega) - \mathrm{Lm}_{asympt}(\omega)$$

$$\Delta\mathrm{Lm}(1/T) = 20\lg k - 20\lg\sqrt{1+(T/T)^2} -$$

$$- \big(20\lg k - 20\lg(T/T)\big) = -20\lg\sqrt{2} = -3{,}03\,[\mathrm{dB}]$$



Rys. 11.

$$\Delta\mathrm{Lm}\left(\frac{1}{T}\right) = -3{,}03\,[\mathrm{dB}]$$

$$\Delta\mathrm{Lm}\left(\frac{1}{2T}\right) = -20\lg\sqrt{1+\left(\frac{T}{2T}\right)^2} + 0 =$$

$$= -20\lg\frac{\sqrt{5}}{2} = -0{,}97\,[\mathrm{dB}]$$

$$\Delta\mathrm{Lm}\left(\frac{2}{T}\right) = -20\lg\sqrt{1+\left(\frac{2T}{T}\right)^2} + 20\lg\frac{2T}{T} =$$

$$= -20\lg\sqrt{5} + 20\lg 2 = -0{,}97\,[\mathrm{dB}]$$

Zalety charakterystyk logarytmicznych

$$G(j\omega) = G_1(j\omega)G_2(j\omega)G_3(j\omega)$$

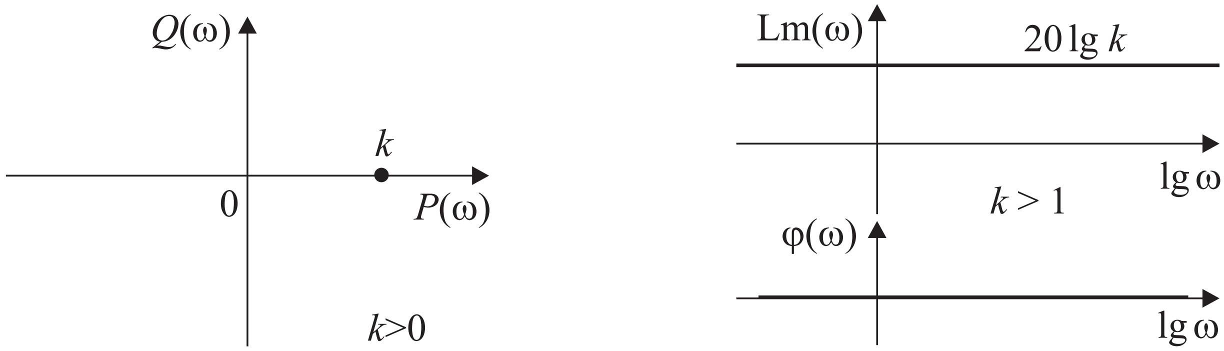
$$\mathrm{Lm}(\omega) = \mathrm{Lm}_1(\omega) + \mathrm{Lm}_2(\omega) + \mathrm{Lm}_3(\omega)$$

$$\varphi(\omega) = \varphi_1(\omega) + \varphi_2(\omega) + \varphi_3(\omega)$$

Przykład (element proporcjonalny)

$$G(j\omega) = k = k + j0$$

$$\mathrm{Lm}(\omega) = 20\lg k, \qquad \varphi(\omega) = \begin{cases} 0 & \text{dla} \quad k > 0 \\ -\pi & \text{dla} \quad k < 0 \end{cases}$$



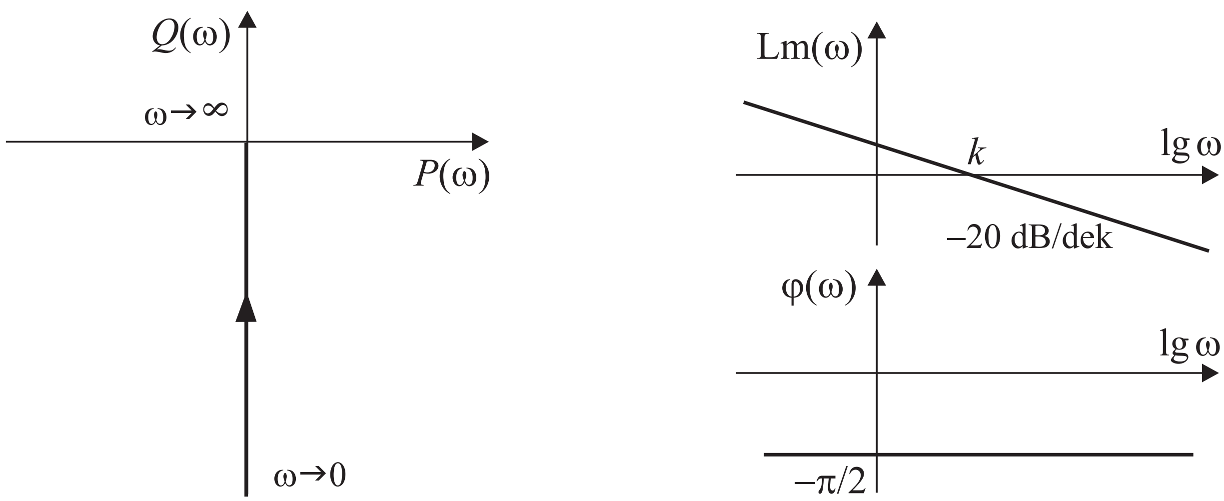
(a) Rys. 12. (b)

Przykład (element całkujący idealny)

$$G(j\omega) = \frac{k}{j\omega} = -j\frac{k}{\omega}, \qquad k > 0, \qquad k \ \left[\frac{\cdot}{\mathrm{s}}\right]$$

$$G(j0) \to 0 - j\infty, \qquad G(j\infty) \to 0 + j0$$

$$\mathrm{Lm}(\omega) = 20\lg k - 20\lg\omega, \quad \varphi(\omega) = -\pi/2$$



(a) Rys. 13. (b)

Przykład (element całkujący rzeczywisty)

$$G(j\omega) = \frac{k}{j\omega(1+j\omega T)} = \frac{k(-\omega^2 T - j\omega)}{\omega^2(1+\omega^2T^2)} =$$

$$= \frac{-kT}{1+\omega^2T^2} + j\frac{-k}{\omega(1+\omega^2T^2)}, \qquad k > 0$$

$$G(j0) \to -kT - j\infty, \qquad G(j\infty) \to 0 + j0$$

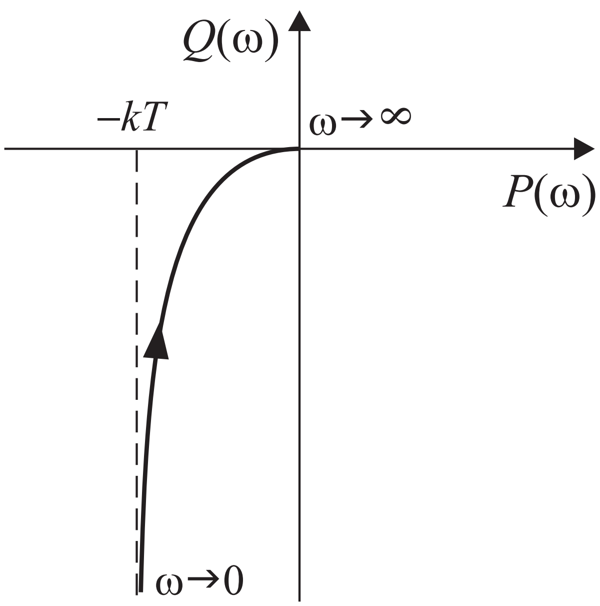
$$G(j\omega) = G_1(j\omega)G_2(j\omega) = \frac{k}{j\omega}\frac{1}{1+j\omega T},$$

$$\mathrm{Lm}(\omega) = \mathrm{Lm}_1(\omega) + \mathrm{Lm}_2(\omega),$$

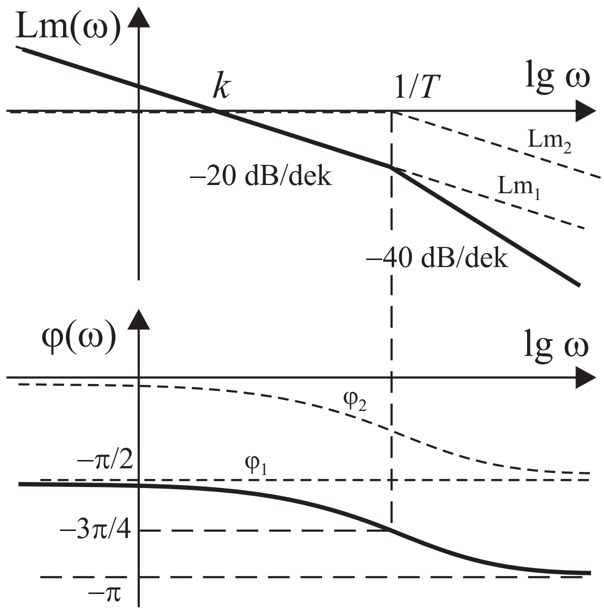
$$\mathrm{Lm}_1(\omega) = 20\lg k - 20\lg(\omega),$$

$$\mathrm{Lm}_2(\omega) = -20\lg\sqrt{1+\omega^2T^2} \approx \begin{cases} 0, & \omega < 1/T \\ -20\lg(\omega T), & \omega \geqslant 1/T \end{cases}$$

$$\varphi(\omega) = \varphi_1(\omega) + \varphi_2(\omega) = -\pi/2 - \mathrm{arctg}(\omega T)$$



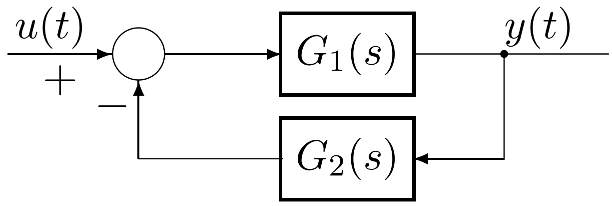
(a)



(b)

Rys. 14.

4. Stabilność ciągłych liniowych układów dynamicznych



Rys. 15.

$$G_1(s) = \frac{L_1(s)}{M_1(s)}, \qquad G_2(s) = \frac{L_2(s)}{M_2(s)},$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_1(s)}{1 + G_1(s)G_2(s)} = \tag{15}$$

$$= \frac{L_1(s)M_2(s)}{M_1(s)M_2(s) + L_1(s)L_2(s)} = \frac{L(s)}{M(s)},$$

$$M(s) = 0 - \text{ równanie charakterystyczne}$$

$$G(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{a_n s^n + \cdots + a_1 s + a_0} = k\frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - s_1)(s - s_2)\ldots(s - s_n)} \tag{16}$$

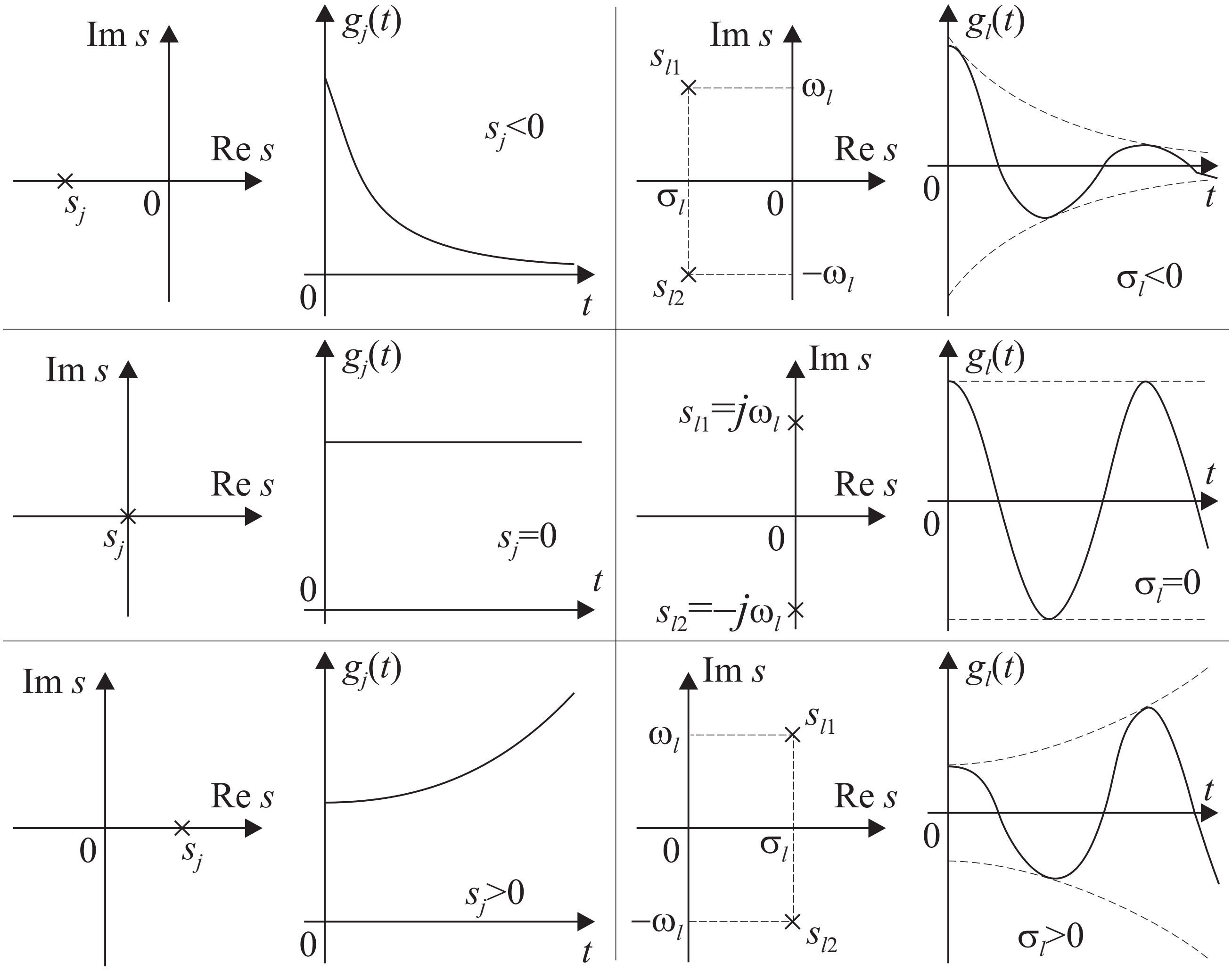
$z_1, \ldots z_m$ – zera transm., $s_1, \ldots s_n$ – bieguny, $k = \dfrac{b_m}{a_n}$

$$G(s) = \frac{k\prod_{i=1}^{m}(s - z_i)}{\prod_{j=1}^{q}(s - s_j)\prod_{l=1}^{r}\left(s^2 + 2\sigma_l s + (\sigma_l^2 + \omega_l^2)\right)}, \tag{17}$$

$q + 2r = n$, bieguny pojedyncze

$$g(t) = \left(\sum_{j=1}^{q} A_j e^{s_j t} + \sum_{l=1}^{r} \frac{B_l}{\omega_l} e^{\sigma_l t}\sin(\omega_l t + \theta_l)\right)\mathbb{1}(t) \tag{18}$$

A_j, B_l, θ_l są stałymi zależnymi od $k, z_i, s_j, \sigma_l, \omega_l$



Rys. 16.

jeżeli biegun s_j jest p-krotny, to wtedy:

$$\mathcal{L}^{-1}\left\{\frac{A_j}{(s-s_j)^p}\right\} = \frac{A_j}{(p-1)!}t^{p-1}e^{s_j t}\mathbb{1}(t) \quad \text{(ogranicz. ampl. dla } s_j<0\text{)} \tag{19}$$

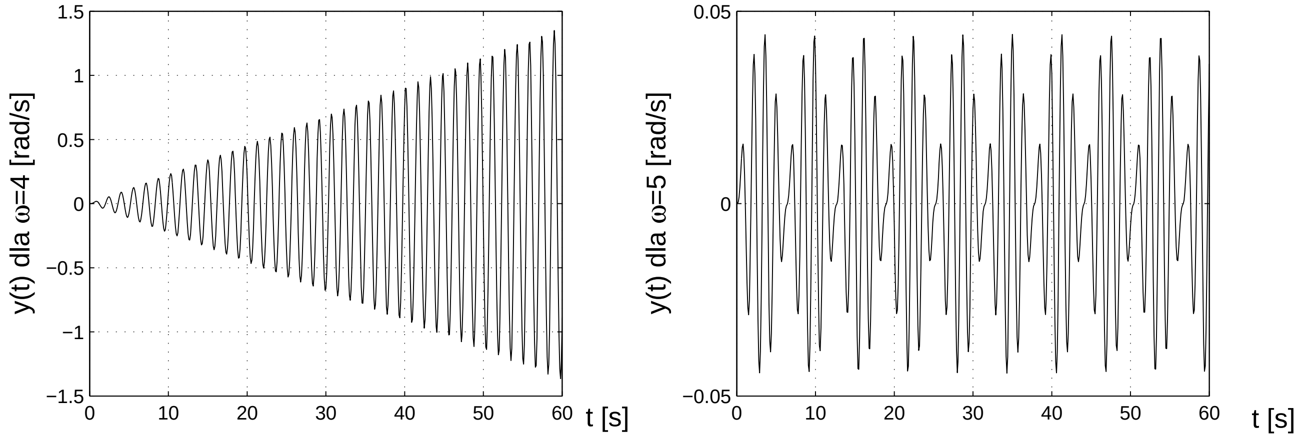
$u(t) = A\sin(\omega_l t)\mathbb{1}(t)$

Przykład

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{(s+10)(s^2+16)} \quad \rightarrow \quad \omega_l = 4 \text{ [rad/s]}$$

$$u(t) = \sin(4t)\mathbb{1}(t), \qquad y(t) = ?$$

```
omega = 4; % [rad/s]
G = tf(2,conv([1 10],[1 0 16]));
t = 0:0.1:60; u = sin(omega*t);
y = lsim(G,u,t); plot(t,y)
```



Rys. 17.