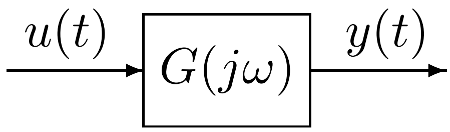
1. Odpowiedź ciągłego układu liniowego na wymuszenie sinusoidalne

$$G(j\omega) = G(s)|_{s=j\omega} \tag{1}$$



Rys. 1.

$$y(t) = y_p(t) + y_u(t), \qquad \lim_{t\to\infty} y_p(t) = 0 \tag{2}$$

$$u(t) = A\sin\omega t \cdot \mathbb{1}(t) \Rightarrow$$
$$\Rightarrow y(t) = y_u(t) = A|G(j\omega)|\sin\big(\omega t + \varphi(\omega)\big) \cdot \mathbb{1}(t) \tag{3}$$

Przykład

$$G(s) = \frac{2}{s^2+3s+2} = \frac{2}{(s+1)(s+2)} = \frac{1}{(s+1)(0{,}5s+1)}$$

$u(t) = 8\sin 2t \cdot \mathbb{1}(t) \rightarrow$ wyznaczyć przebieg $y_u(t)$

$$G(s)|_{s=j2} = \frac{2}{(j2)^2+3(j2)+2} = \frac{2}{-2+j6} = 0{,}316\, e^{-j108{,}4^\circ}$$

$$y_u(t) = 8\cdot 0{,}316\sin(2t-108{,}4^\circ)\cdot\mathbb{1}(t) =$$
$$= 2{,}528\sin(2t-108{,}4^\circ)\cdot\mathbb{1}(t)$$

$T_1 = 1[s],\ T_2 = 0{,}5[s] \ \Rightarrow\ y(t) = y_u(t)$ po ok. $4[s]$

$y_p(t) = (k_1e^{-t} + k_2e^{-2t})\cdot\mathbb{1}(t) \rightarrow 0$ przy $t\rightarrow\infty$

2. Charakterystyka amplitudowo-fazowa (wykres Nyquista)

$$G(s)=\frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}, \quad m \leqslant n \qquad (4)$$

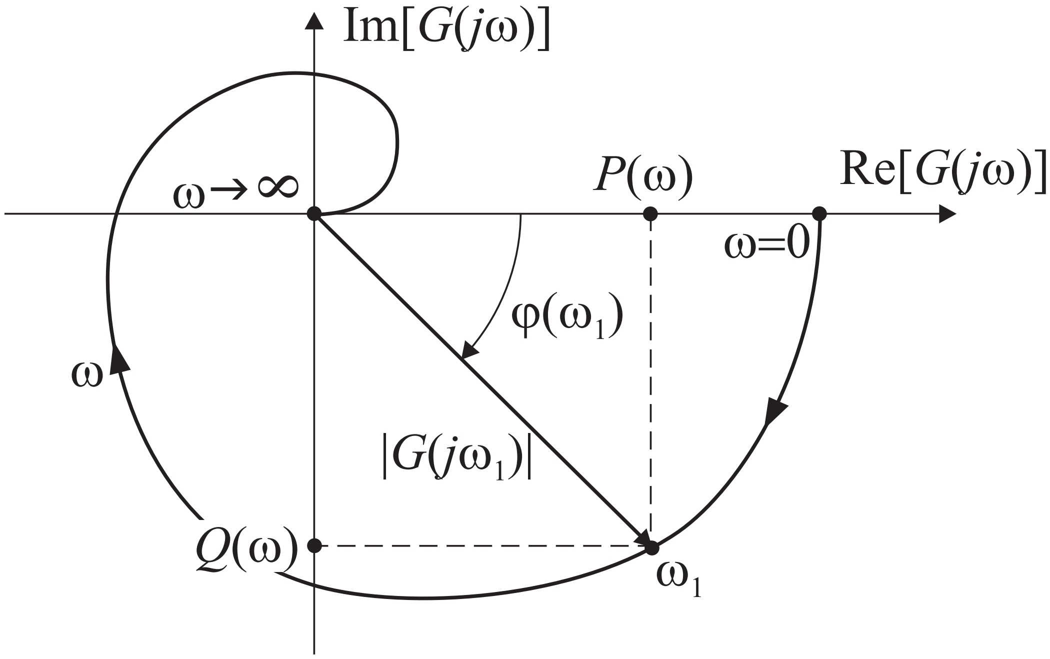
$$G(j\omega) = G(s)|_{s=j\omega} = \qquad (5)$$

$$= \frac{b_m (j\omega)^m + b_{m-1}(j\omega)^{m-1} + \cdots + b_1 (j\omega) + b_0}{a_n (j\omega)^n + a_{n-1}(j\omega)^{n-1} + \cdots + a_1 (j\omega) + a_0}$$

$$G(j\omega) = P(\omega) + jQ(\omega) = |G(j\omega)|e^{j\varphi(\omega)} \qquad (6)$$

$$P(\omega) = \mathrm{Re}[G(j\omega)], \qquad |G(j\omega)| = \sqrt{P^2(\omega) + Q^2(\omega)},$$

$$Q(\omega) = \mathrm{Im}[G(j\omega)], \qquad \varphi(\omega) = \mathrm{arctg}\frac{Q(\omega)}{P(\omega)}.$$



Rys. 2. $(0 \leqslant \omega < \infty)$

$$u(t) = A\sin\omega_1 t \Rightarrow y(t) = A|G(j\omega_1)|\sin\big(\omega_1 t + \varphi(\omega_1)\big)$$

$$G(j\omega) = \frac{a(\omega) + jb(\omega)}{c(\omega) + jd(\omega)} = \frac{(a + jb)(c - jd)}{c^2 + d^2} = \tag{7}$$
$$= \frac{ac + bd}{c^2 + d^2} + j\frac{bc - ad}{c^2 + d^2} = P(\omega) + jQ(\omega),$$

$$a(\omega) = b_0 - b_2\omega^2 + b_4\omega^4 - b_6\omega^6 + \ldots,$$

$$b(\omega) = b_1\omega - b_3\omega^3 + b_5\omega^5 - b_7\omega^7 + \ldots,$$

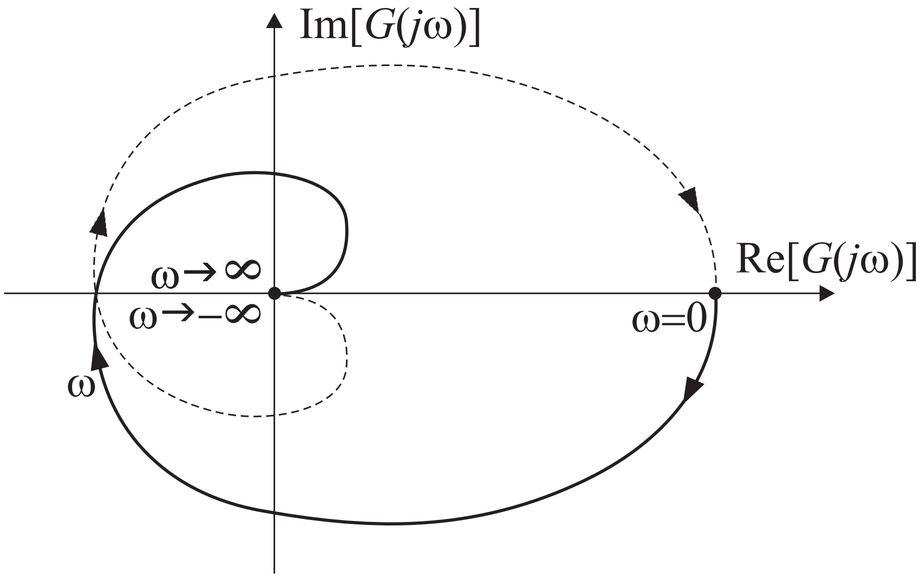
$$c(\omega) = a_0 - a_2\omega^2 + a_4\omega^4 - a_6\omega^6 + \ldots,$$

$$d(\omega) = a_1\omega - a_3\omega^3 + a_5\omega^5 - a_7\omega^7 + \ldots,$$

$$G(-j\omega) = P(-\omega) + jQ(-\omega) = \frac{a(\omega) - jb(\omega)}{c(\omega) - jd(\omega)} = \tag{8}$$
$$= \frac{ac + bd}{c^2 + d^2} + j\frac{ad - bc}{c^2 + d^2} = P(\omega) - jQ(\omega),$$

$P(-\omega) = P(\omega)$ – funkcja parzysta,

$Q(-\omega) = -Q(\omega)$ – funkcja nieparzysta



Rys. 3. $(-\infty < \omega < \infty)$

Przykład (element inercyjny 1-go rzędu)

$$G(j\omega) = \frac{k}{1+j\omega T} = \frac{k(1-j\omega T)}{1+\omega^2T^2} =$$

$$= \frac{k}{1+\omega^2T^2} - j\frac{k\omega T}{1+\omega^2T^2} = P(\omega) + jQ(\omega)$$

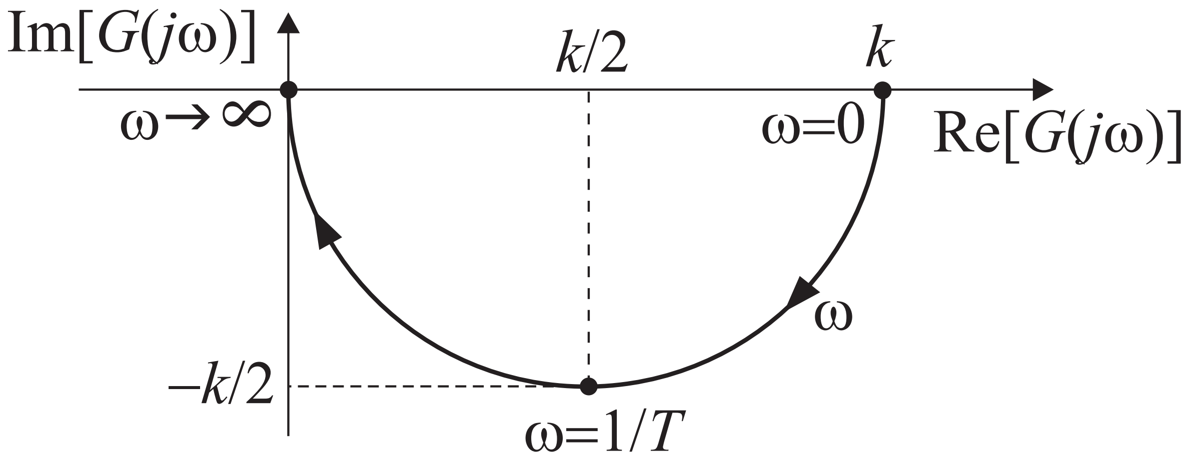
$$\omega > 0 \quad \rightarrow \quad P > 0,\ Q \leqslant 0 \quad \rightarrow \quad \text{IV ćw.}$$

$$P = \frac{k}{1+\omega^2T^2} \quad \rightarrow \quad 1+\omega^2T^2 = \frac{k}{P} \quad \rightarrow \quad \omega^2T^2 = \frac{k-P}{P}$$

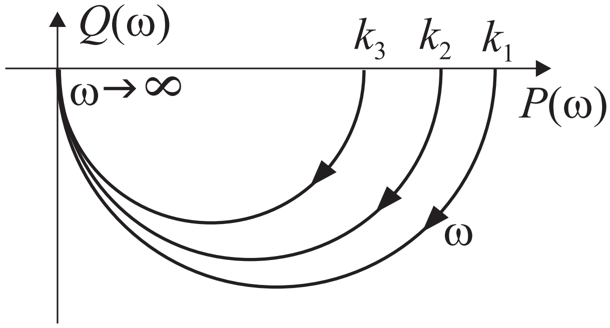
$$Q^2 = \frac{k^2\omega^2T^2}{(1+\omega^2T^2)^2} = \frac{k^2\frac{k-P}{P}}{k^2/P^2} = P(k-P)$$

$$Q^2 + P^2 - kP = 0 \ \rightarrow \quad P^2 - kP + \frac{k^2}{4} + Q^2 = \frac{k^2}{4}$$

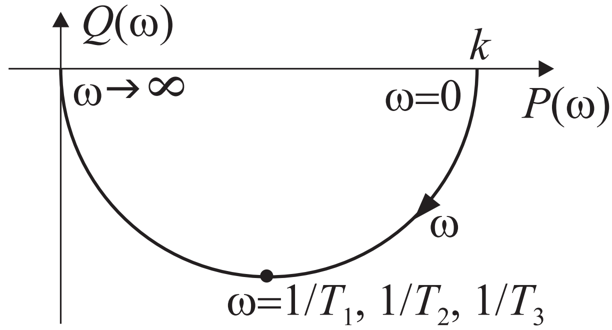
$$\left(P - \frac{k}{2}\right)^2 + Q^2 = \left(\frac{k}{2}\right)^2, \qquad Q \leqslant 0$$



Rys. 4.



(a) $k = \text{var}, k_3 < k_2 < k_1$

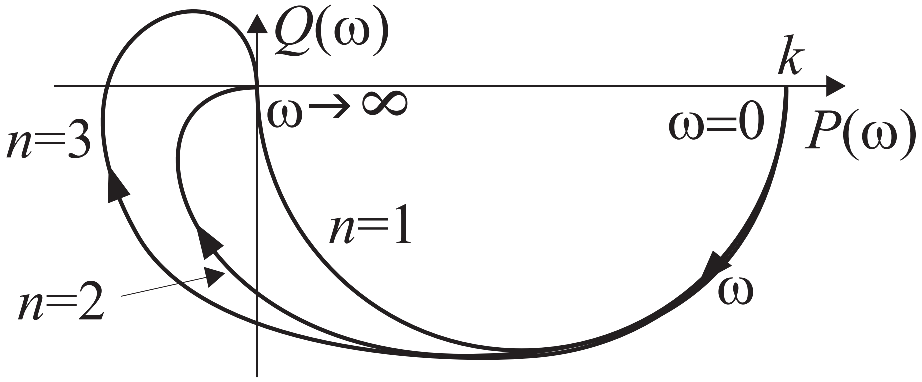


(b) $T = \text{var}$

Rys. 5.

Przykład (element inercyjny n-tego rzędu)

$$G(j\omega) = \frac{k}{(1 + j\omega T)^n}, \qquad n = 1, 2, 3, \ldots$$



Rys. 6.

Własności charakterystyki a-f

1. $$G(j\omega) = \frac{b_m(j\omega)^m + \cdots + b_1(j\omega) + b_0}{a_n(j\omega)^n + \cdots + a_1(j\omega) + a_0} = \qquad (m \leqslant n)$$

$$= \frac{b_m(j\omega)^{m-n} + \cdots + b_1(j\omega)^{1-n} + b_0(j\omega)^{-n}}{a_n + \cdots + a_1(j\omega)^{1-n} + a_0(j\omega)^{-n}} \qquad (9)$$

$G(j\omega) \to b_m/a_n + j0$ przy $j\omega \to \infty$ gdy $m = n$

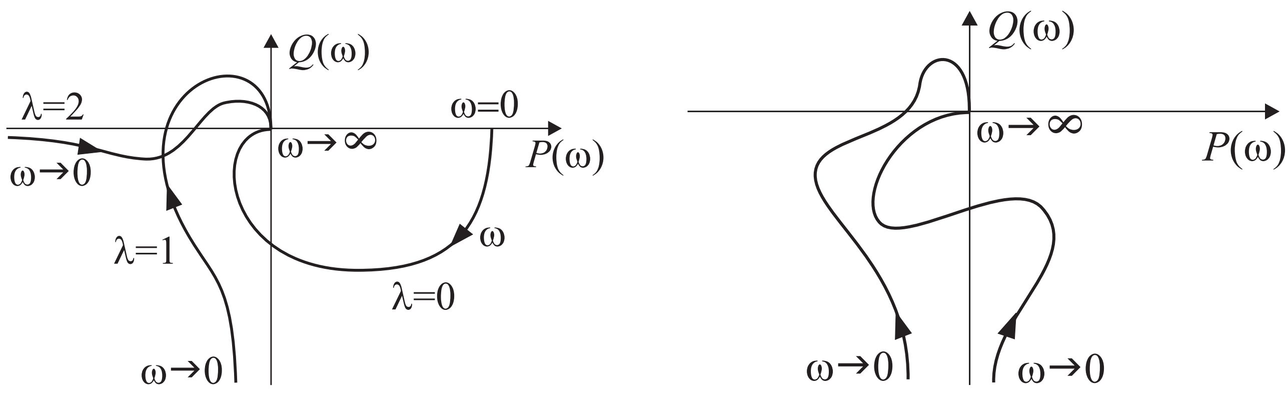
$G(j\omega) \to 0 + j0$ przy $j\omega \to \infty$ gdy $m < n$

2. $$G(j\omega) = \frac{k(1 + j\omega T_a)(1 + j\omega T_b)\ldots}{(j\omega)^\lambda(1 + j\omega T_1)(1 + j\omega T_2)\ldots} \qquad (10)$$

(a) $\lambda = 0:\quad G(j0) = b_0/a_0 + j0$

(b) $\lambda = 1:\quad \dfrac{1}{j\omega} \to \Delta\varphi = -\dfrac{\pi}{2}$ przy $0 \leqslant \omega < \infty$

(c) $\lambda = 2:\quad \dfrac{1}{(j\omega)^2} \to \Delta\varphi = -\pi$ przy $0 \leqslant \omega < \infty$



(a) (b)

Rys. 7.

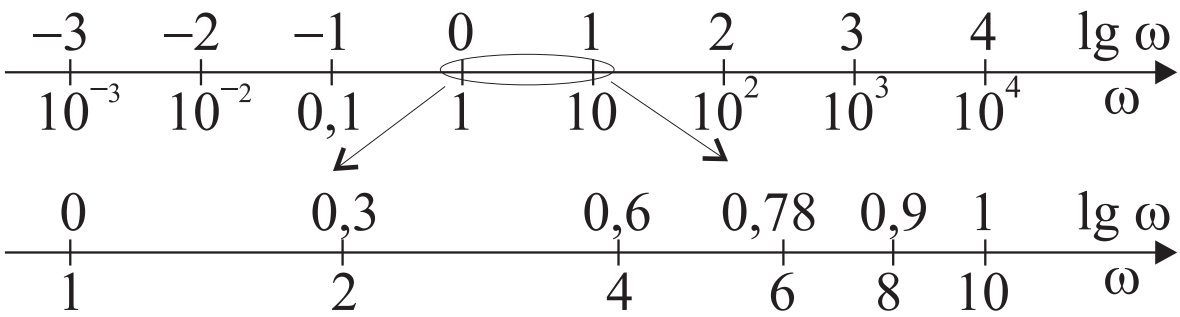
3. Charakterystyki logarytmiczne (wykresy Bodego)

$$\mathrm{Lm}(\omega) = 20\lg|G(j\omega)| \qquad \text{[dB = decybel]} \tag{11}$$

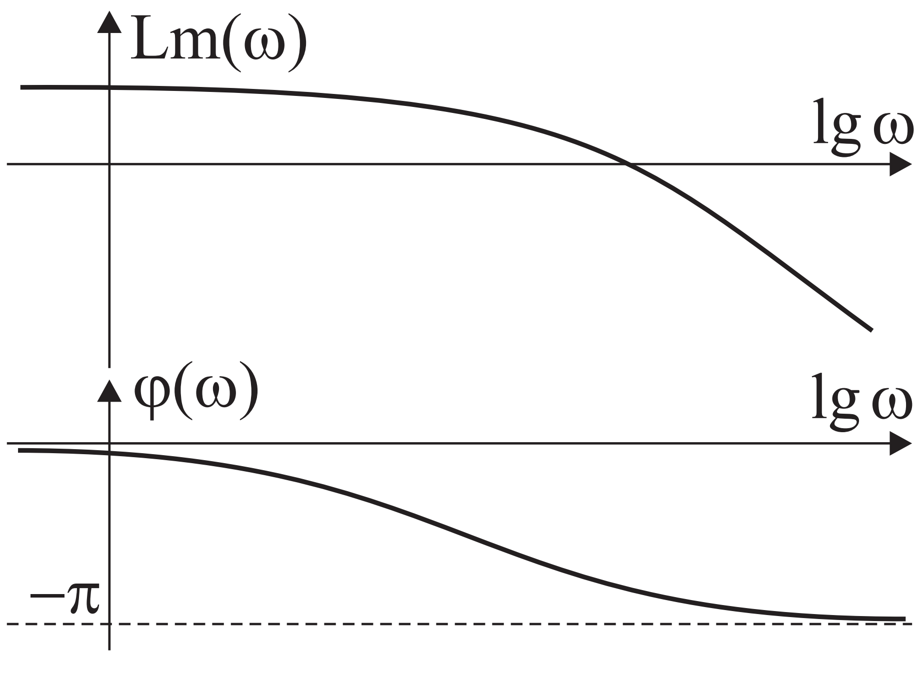
$$\big(\mathrm{Lm}(\omega) = 1\text{[dB]} \rightarrow 20\lg|G(j\omega)| = 1 \rightarrow$$

$$\rightarrow |G(j\omega)| = 10^{1/20} \approx 1{,}22 \rightarrow y_{\max}/u_{\max} \approx 1{,}22\big)$$

$$\varphi(\omega) = \arg(G(j\omega)) \tag{12}$$



Rys. 8.



Rys. 9.