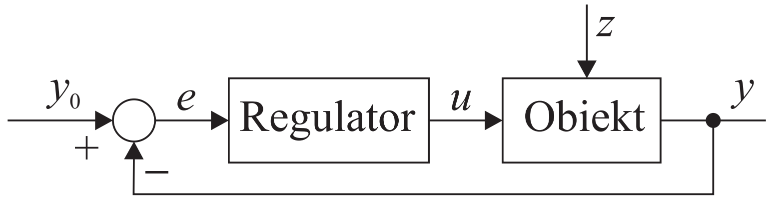
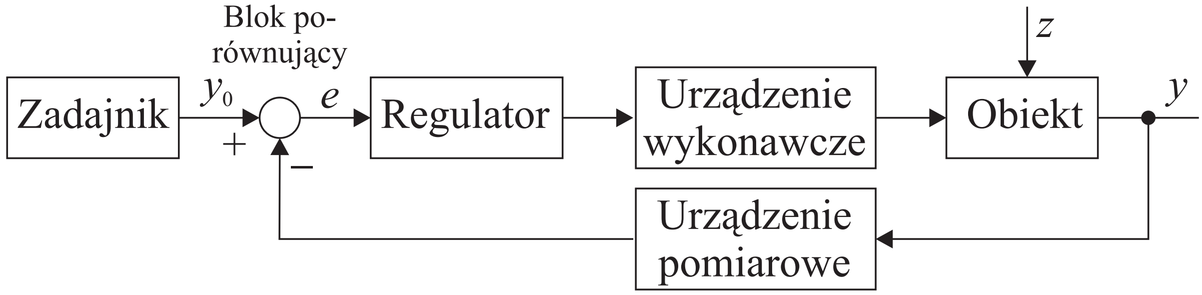
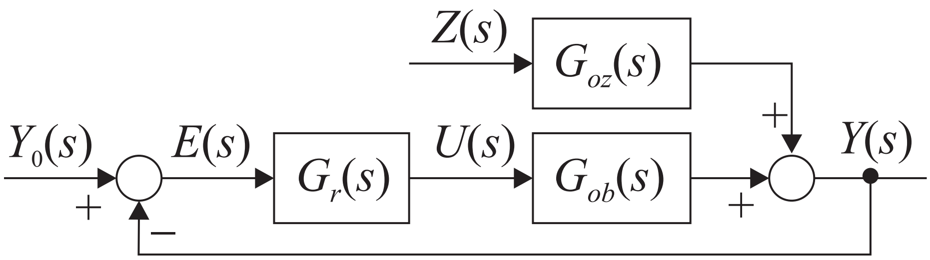
# 18. Układ regulacji automatycznej




Rys. 42 $e(t) = y_0(t) - y(t), \quad E(s) = Y_0(s) - Y(s)$




Rys. 43




Rys. 44 $G_{oz}(s) = 1$ – zakł. na wyjściu, $G_{oz}(s) = G_{ob}(s)$ – zakł. na wejściu

$$Y(s) = G_r(s)G_{ob}(s)E(s) + G_{oz}(s)Z(s) = G_o(s)[Y_0(s) - Y(s)] + G_{oz}(s)Z(s)$$

$G_o(s) = G_r(s)G_{ob}(s)$ – transmitancja układu otwartego

$$[1 + G_o(s)]Y(s) = G_o(s)Y_0(s) + G_{oz}(s)Z(s)$$

$$Y(s) = \frac{G_o(s)}{1 + G_o(s)}Y_0(s) + \frac{G_{oz}(s)}{1 + G_o(s)}Z(s) = G(s)Y_0(s) + G_z(s)Z(s) \tag{25}$$

$G(s) = \left.\frac{Y(s)}{Y_0(s)}\right|_{Z(s)\equiv 0} = \frac{G_o(s)}{1 + G_o(s)}$ – transm. układu zamkniętego

$G_z(s) = \left.\frac{Y(s)}{Z(s)}\right|_{Y_0(s)\equiv 0} = \frac{G_{oz}(s)}{1 + G_o(s)}$ – transmitancja zakłóceniowa

$$E(s) = Y_0(s) - Y(s) = \left(1 - \frac{G_o(s)}{1+G_o(s)}\right) Y_0(s) - \frac{G_{oz}(s)}{1+G_o(s)} Z(s)$$

$$E(s) = \frac{1}{1+G_o(s)} Y_0(s) - \frac{G_{oz}(s)}{1+G_o(s)} Z(s) = G_e(s) Y_0(s) - G_{ez}(s) Z(s) \tag{26}$$

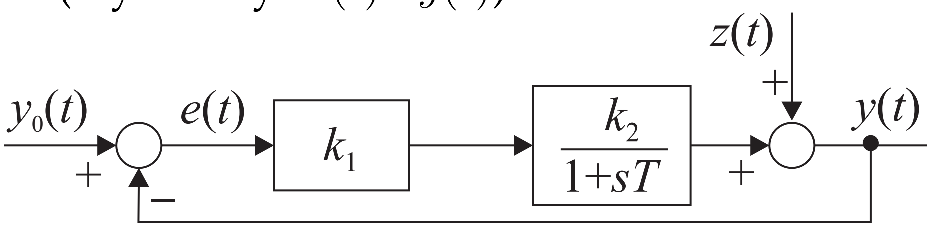
$G_e(s) = \left.\frac{E(s)}{Y_0(s)}\right|_{Z(s)\equiv 0} = \frac{1}{1+G_o(s)}$ – transmitancja uchybowa

$$G_{ez}(s) = \left.\frac{E(s)}{Z(s)}\right|_{Y_0(s)\equiv 0} = \frac{G_{oz}(s)}{1+G_o(s)} = G_z(s) = \left.\frac{Y(s)}{Z(s)}\right|_{Y_0(s)\equiv 0}$$

Zauważmy, że

$$G_e(s) = \frac{1}{1+G_o(s)} = \frac{1+G_o(s)-G_o(s)}{1+G_o(s)} = 1 - G(s) \tag{27}$$

**Przykład** (wyznaczyć $e(t)$ i $y(t)$)




Rys. 45 $y_0(t) = \mathbb{1}(t) \rightarrow Y_0(s) = 1/s, \quad z(t) = 0$

$$G_o(s) = G_r(s) G_{ob}(s) = k_1 \frac{k_2}{1+sT} = \frac{k}{1+sT}, \quad k = k_1 k_2$$

$$E(s) = \frac{1}{1+\frac{k}{1+sT}} Y_0(s) - \frac{1}{1+\frac{k}{1+sT}} Z(s) = \frac{1+sT}{1+k+sT}[Y_0(s) - Z(s)]$$

$$G_e(s) = \frac{1+sT}{1+k+sT}, \qquad G(s) = 1 - G_e(s) = \frac{k}{1+k+sT}$$

$$E(s) = G_e(s) Y_0(s) = \frac{1+sT}{(1+k+sT)s} = \frac{1+sT}{T(\frac{1+k}{T}+s)s}$$

$$e(t) = \mathcal{L}^{-1}[E(s)] = \mathcal{L}^{-1}\left[\frac{1+sT}{T(\frac{1+k}{T}+s)s}\right] = \left(\lim_{s\to 0}\frac{1+sT}{1+k+sT}e^{st}+\right.$$

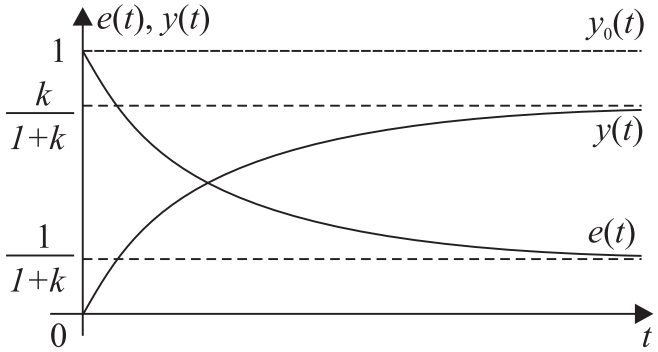
$$\left.+\lim_{s\to -\frac{1+k}{T}}\frac{1+sT}{Ts}e^{st}\right)\mathbb{1}(t) = \left(\frac{1}{1+k}+\frac{1-\frac{1+k}{T}T}{-\frac{1+k}{T}T}e^{-\frac{1+k}{T}t}\right)\mathbb{1}(t) =$$

$$= \left(\frac{1}{1+k}+\frac{k}{1+k}e^{-\frac{1+k}{T}t}\right)\mathbb{1}(t)$$

$$e(t) = y_0(t) - y(t)$$

$$y(t) = y_0(t) - e(t) = \left(1-\frac{1}{1+k}-\frac{k}{1+k}e^{-\frac{1+k}{T}t}\right)\mathbb{1}(t) =$$

$$= \frac{k}{1+k}\left(1-e^{-\frac{1+k}{T}t}\right)\mathbb{1}(t)$$




Rys. 46 $\quad e_u = \lim_{t\to\infty} e(t) = \frac{1}{1+k}$

$$e_u = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \not{s}\frac{1+sT}{(1+k+sT)\not{s}} = \frac{1}{1+k}$$

$$G(s) = \frac{k}{1+k+sT} = \frac{\frac{k}{1+k}}{1+s\frac{T}{1+k}} = \frac{k_z}{1+sT_z}$$

$$k_z = \frac{k}{1+k}, \qquad T_z = \frac{T}{1+k}$$

## 19. Klasyfikacja URA ze wzgl. na postać sygn. wejściowego

a) regulacja stałowartościowa $y_0(t) = \text{const}$

b) regulacja programowa

c) regulacja nadążna

## 20. Zadania układów regulacji

a) zadanie nadążania $e(t) \approx 0$

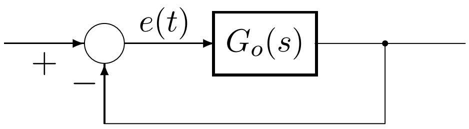
b) zadanie przestawiania

c) zadanie kompensacji zakłóceń

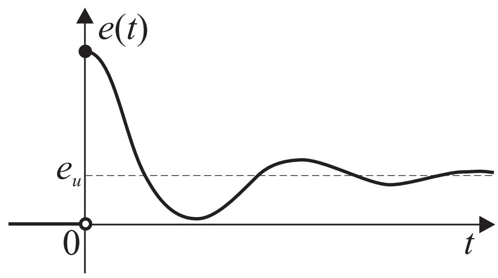
## 21. Wskaźniki jakości procesu regulacji

a) wartość uchybu ustalonego $e_u$ $\quad e(t) = e_u + e_p(t)$

$$\lim_{t\to\infty} e_p(t) = 0 \quad \Rightarrow \quad \lim_{t\to\infty} e(t) = e_u = \lim_{s\to 0} sE(s)$$



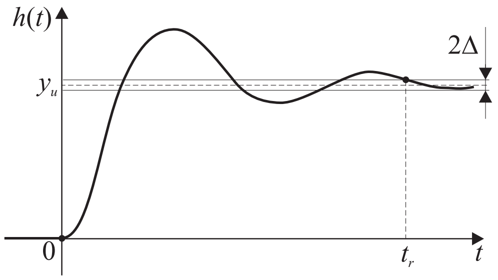

(a)



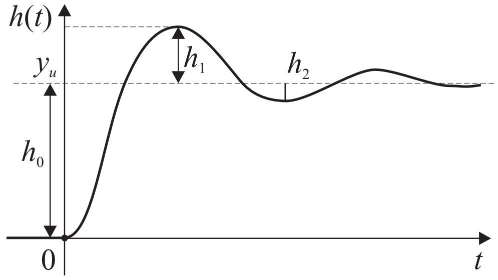

(b)

Rys. 47

b) czas regulacji $t_r$




(a)




(b)

Rys. 48

c) przeregulowanie $\varkappa$: $\quad \varkappa = \dfrac{h_1}{h_0} \cdot 100\% \quad \left(\dfrac{h_3}{h_2} = \dfrac{h_2}{h_1} = \dfrac{h_1}{h_0}\right)$

## 22. Stabilność ciągłych układów liniowych


Rys. 49

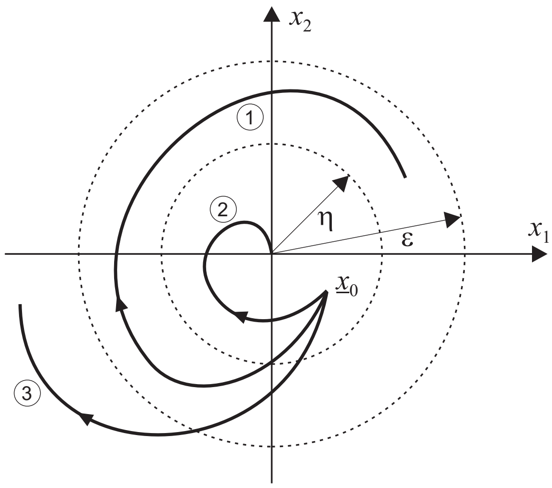
$$\underline{\dot{x}} = \underline{F}(\underline{x}) \tag{28}$$

$\underline{x} = \underline{0}$ – punkt równowagi układu, tzn. $\underline{F}(\underline{0}) = \underline{0}$

$\underline{x}(0) = \underline{x}_0$ dla $t = 0$

$$\underline{x} = \underline{0} \qquad \bigwedge_{t>0} \bigwedge_{\varepsilon>0} \bigvee_{\eta>0} \|\underline{x}_0\| < \eta \Rightarrow \|\underline{x}(t)\| < \varepsilon \tag{29}$$

$$\|\underline{x}\| = \sqrt{\sum_{i=1}^{n} x_i^2} \tag{30}$$




Rys. 50

$$\lim_{t\to\infty} \underline{x}(t) = \underline{0} \tag{31}$$

## 23. Warunek konieczny i dostateczny stabilności ukł. ciągłych

$$G(s) = \frac{L(s)}{M(s)} = \frac{b_m s^m + \cdots + b_1 s + b_0}{a_n s^n + \cdots + a_1 s + a_0} = k\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-s_1)(s-s_2)\ldots(s-s_n)}$$

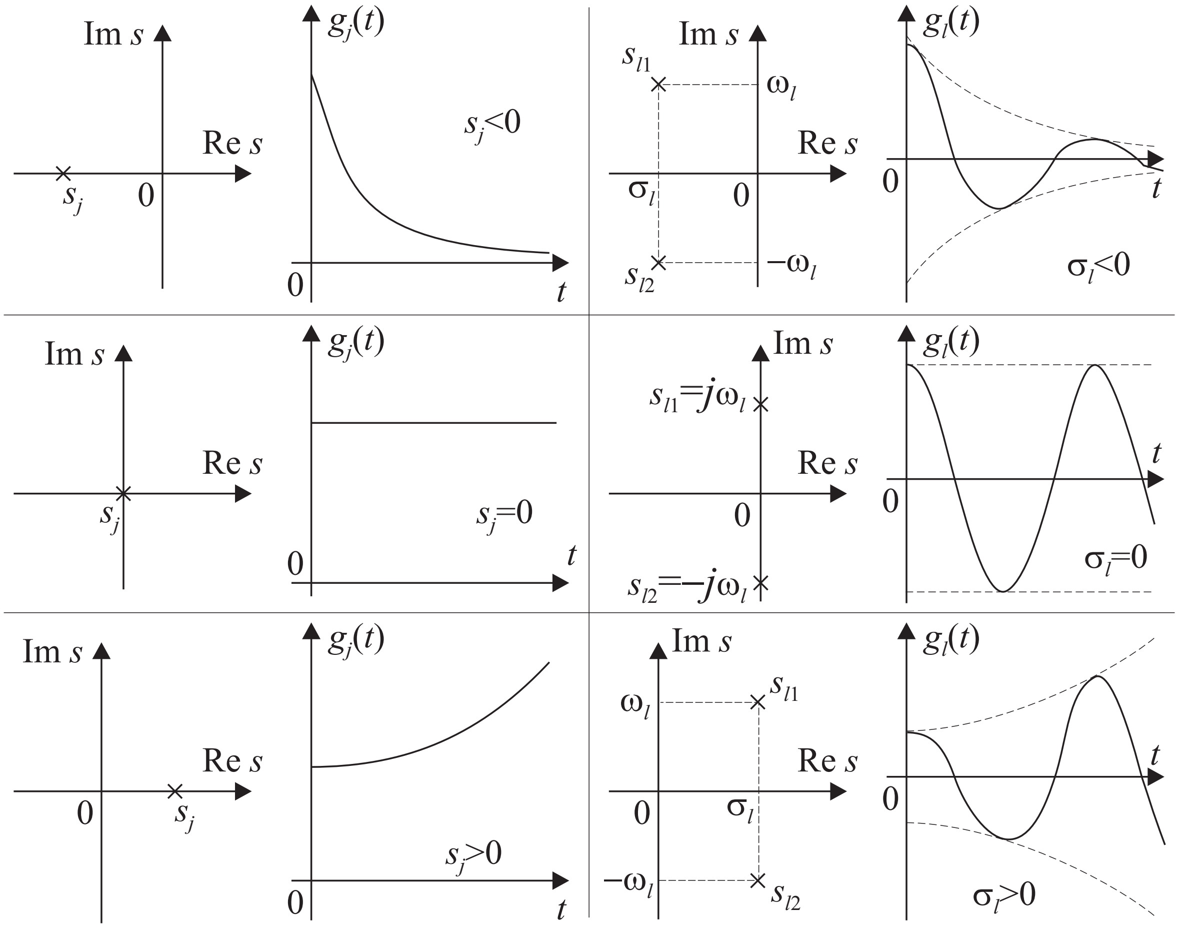
$z_1, \ldots z_m$ – zera transm., $s_1, \ldots s_n$ – bieguny, $k = b_m/a_n$ (32)

$$G(s) = \frac{k\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{q}(s-s_j)\prod_{l=1}^{r}[s^2+2\sigma_l s+(\sigma_l^2+\omega_l^2)]}, \quad (33)$$

$q + 2r = n$, bieguny pojedyncze $s_j$ lub $s_l = \sigma_l + j\omega_l$

$$g(t) = \mathcal{L}^{-1}[G(s)] = \left[\sum_{j=1}^{q} A_j e^{s_j t} + \sum_{l=1}^{r} \frac{B_l}{\omega_l} e^{\sigma_l t}\sin(\omega_l t + \theta_l)\right]\mathbb{1}(t) \quad (34)$$

$A_j, B_l, \theta_l$ są stałymi zależnymi od $k, z_i, s_j, \sigma_l, \omega_l$




Rys. 51

$G(s) \quad \text{Re}(s_i) < 0, i = 1, \dots n$

$$G(s) = \frac{L(s)}{M(s)} \quad \rightarrow \quad M(s) = a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0 = 0$$

## 24. Kryterium stabilności Hurwitza

$$M(s) = a_n s^n + a_{n-1} s^{n-1} + \dots a_1 s + a_0 = 0 \tag{35}$$

a) $a_0, a_1, \dots a_n > 0$

b) $\Delta_n, \Delta_{n-1}, \dots \Delta_2, \Delta_1$

$$\Delta_n = \underbrace{\begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \dots & 0 \\ a_n & a_{n-2} & a_{n-4} & \dots & 0 \\ 0 & a_{n-1} & a_{n-3} & \dots & 0 \\ 0 & a_n & a_{n-2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}}_{n} \left.\vphantom{\begin{vmatrix} a \\ a \\ a \\ a \\ a \end{vmatrix}}\right\} n \tag{36}$$

### Przykład

$$G_o(s) = \frac{k}{s(1 + sT_1)(1 + sT_2)} \quad \rightarrow \quad G(s) = \frac{G_o(s)}{1 + G_o(s)} = \frac{L(s)}{M(s)}$$

$$M(s) = T_1 T_2 s^3 + (T_1 + T_2) s^2 + s + k = a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

$$T_1 T_2 > 0, \qquad T_1 + T_2 > 0, \qquad k > 0$$

$$\Delta_3 = \begin{vmatrix} a_2 & a_0 & 0 \\ a_3 & a_1 & 0 \\ 0 & a_2 & a_0 \end{vmatrix} = \begin{vmatrix} T_1 + T_2 & k & 0 \\ T_1 T_2 & 1 & 0 \\ 0 & T_1 + T_2 & k \end{vmatrix}$$

$$T_1 + T_2 > T_1 T_2 k$$

## 25. Kryterium stabilności Routha

$$
\begin{array}{c|ccccc}
s^n & a_n & a_{n-2} & a_{n-4} & a_{n-6} & \dots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \dots \\
s^{n-2} & b_1 & b_2 & b_3 & \dots & \\
s^{n-3} & c_1 & c_2 & c_3 & \dots & \\
s^{n-4} & d_1 & d_2 & \dots & & \\
\dots & \dots\dots\dots\dots & & & &
\end{array}
\tag{37}
$$

$$
b_1 = \frac{\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}}{-a_{n-1}}, \quad
b_2 = \frac{\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}}{-a_{n-1}}, \quad
b_3 = \frac{\begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix}}{-a_{n-1}}, \quad \dots
$$

$$
c_1 = \frac{\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}}{-b_1}, \quad
c_2 = \frac{\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}}{-b_1}, \quad \dots
$$

$$
d_1 = \frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{-c_1}, \quad
d_2 = \frac{\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}}{-c_1}, \quad \dots
$$

$$
G(s) = \frac{L(s)}{M(s)}, \qquad M(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0
$$

**Przykład**

$$M(s) = (s+2)(s^2 - s + 4) = s^3 + s^2 + 2s + 8 = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 1 & 8 \\ s^1 & -6 & \\ s^0 & 8 & \end{array} \qquad b_1 = -\frac{1}{1}\begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} = -6, \quad c_1 = -\frac{1}{-6}\begin{vmatrix} 1 & 8 \\ -6 & 0 \end{vmatrix} = 8$$

układ niestabilny, 2 bieguny w prawej półpłaszcz. zespolonej

**Przykład**

$$M(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$$

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & \epsilon & 6 & \\ s^2 & -\frac{12}{\epsilon} & 10 & \\ s^1 & 6 & & \\ s^0 & 10 & & \end{array}$$

$$b_1 = -\frac{1}{2}\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 0 \approx \epsilon, \quad b_2 = -\frac{1}{2}\begin{vmatrix} 1 & 11 \\ 2 & 10 \end{vmatrix} = 6,$$

$$c_1 = -\frac{1}{\epsilon}\begin{vmatrix} 2 & 4 \\ \epsilon & 6 \end{vmatrix} \approx -\frac{12}{\epsilon}, \quad c_2 = -\frac{1}{\epsilon}\begin{vmatrix} 2 & 10 \\ \epsilon & 0 \end{vmatrix} = 10,$$

$$d_1 = \frac{\epsilon}{12}\begin{vmatrix} \epsilon & 6 \\ -\frac{12}{\epsilon} & 10 \end{vmatrix} = \frac{\epsilon}{12}\left(10\epsilon + 6\frac{12}{\epsilon}\right) \approx 6, \; e_1 = -\frac{1}{6}\begin{vmatrix} -\frac{12}{\epsilon} & 10 \\ 6 & 0 \end{vmatrix} = 10$$

układ niestabilny, 2 bieguny w prawej półpłaszcz. zespolonej

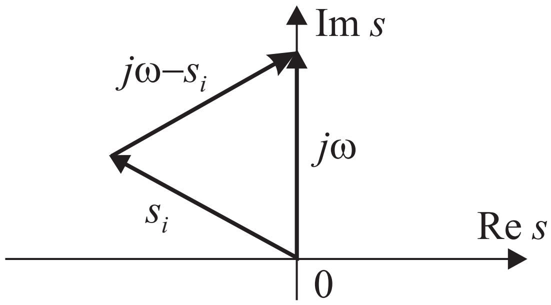
```
  p = [1 2 2 4 11 10];
  roots(p)
ans =
   0.8950 + 1.4561i,   0.8950 - 1.4561i
  -1.2407 + 1.0375i,  -1.2407 - 1.0375i
  -1.3087
```

## 26. Kryterium stabilności Michajłowa (częstotliwościowe)

$$G(s) = \frac{L(s)}{M(s)}, \qquad M(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots a_1 s + a_0 =$$

$$= a_n (s - s_1)(s - s_2) \ldots (s - s_n) \qquad (38)$$

$$M(j\omega) = M(s)\Big|_{s=j\omega} = a_n (j\omega - s_1)(j\omega - s_2) \ldots (j\omega - s_n) \qquad (39)$$

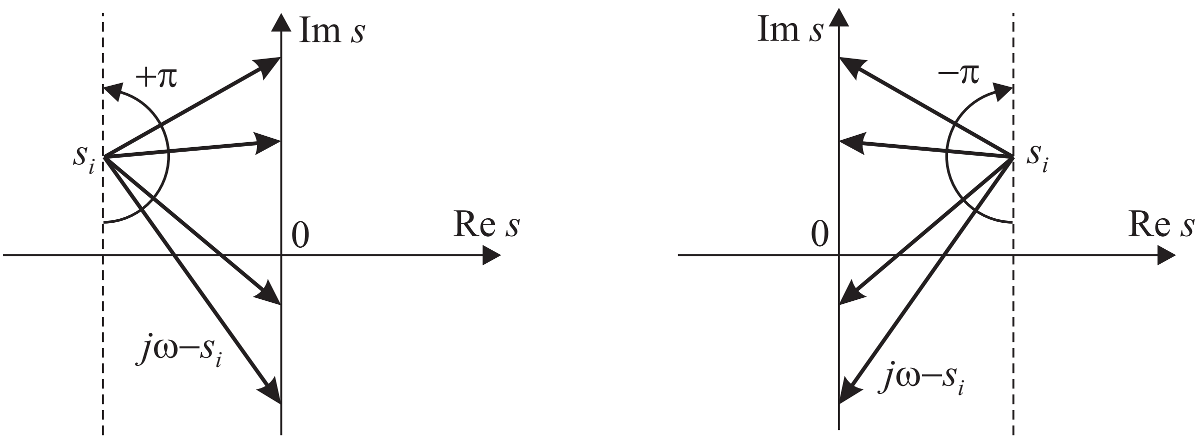



Rys. 52

$$\boxed{Z(j\omega) = |Z(j\omega)| e^{j\varphi(\omega)}, \qquad \varphi(\omega) = \arg Z(j\omega)}$$

$$\mathrm{Re}(s_i) < 0 \quad \Rightarrow \quad \underset{-\infty<\omega<\infty}{\Delta \arg} (j\omega - s_i) = +\pi$$

$$\mathrm{Re}(s_i) > 0 \quad \Rightarrow \quad \underset{-\infty<\omega<\infty}{\Delta \arg} (j\omega - s_i) = -\pi$$




Rys. 53

$$\Delta \arg M(j\omega) = \sum_{i=1}^{n} \Delta \arg (j\omega - s_i) \qquad (40)$$

Jeśli w prawej półpłaszcz. zespolonej $l$ pierwiastków $M(j\omega)$, to:

$$\Delta \underset{-\infty<\omega<\infty}{\arg} M(j\omega) = l(-\pi) + (n-l)\pi = (n-2l)\pi \qquad (41)$$

Jeśli układ asymptotycznie stabilny, tzn. $l = 0$, to:

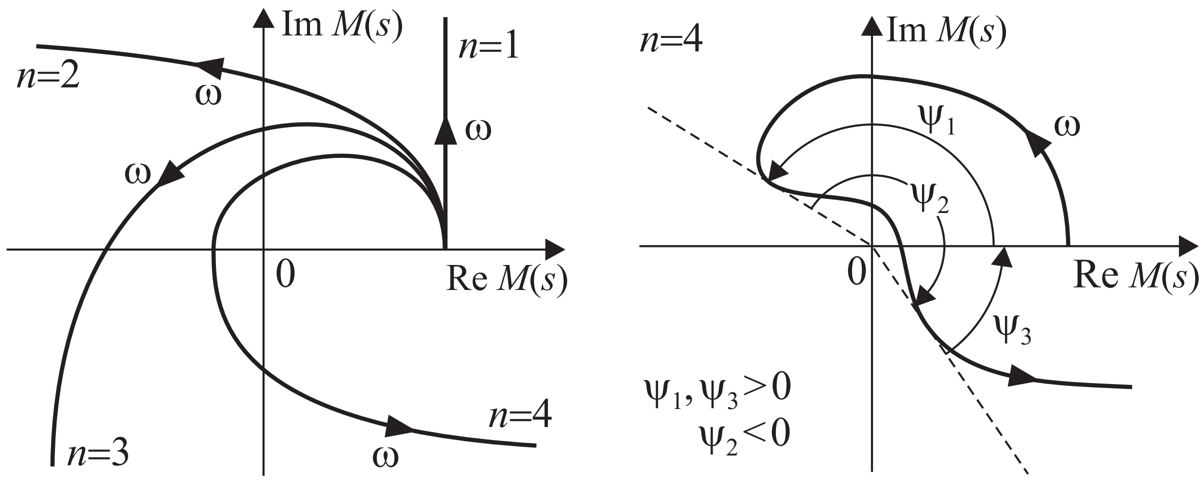
$$\Delta \underset{-\infty<\omega<\infty}{\arg} M(j\omega) = n\pi \qquad (42)$$

$$\mathrm{Re}[M(-j\omega)] = \mathrm{Re}[M(j\omega)], \qquad \mathrm{Im}[M(-j\omega)] = -\mathrm{Im}[M(j\omega)]$$

$$-\infty < \omega < \infty \quad \rightarrow \quad 0 \leqslant \omega < \infty$$

$$\Delta \underset{0\leqslant\omega<\infty}{\arg} M(j\omega) = n\frac{\pi}{2} \qquad (\Rightarrow \ \mathrm{Re}(s_i) < 0, \ \ i = 1, 2, \ldots n) \qquad (43)$$

$$\Delta \underset{0\leqslant\omega<\infty}{\arg} M(j\omega) = (n-2l)\frac{\pi}{2} \qquad (44)$$




(a) układy stabilne (b) układ niestabilny

Rys. 54

układ niestabilny ponieważ

$$\Delta \underset{0\leqslant\omega<\infty}{\arg} M(j\omega) = \psi_1 + \psi_2 + \psi_3 = 0$$

$$(n-2l)\frac{\pi}{2} = 0 \quad \Rightarrow \quad \text{dla} \ \ n = 4 \ \ \text{jest} \ \ l = 2$$