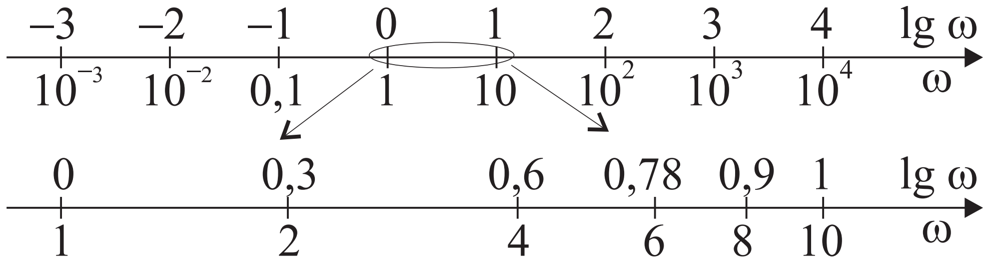
15. Charakterystyki logarytmiczne (wykresy Bodego)

$$\mathrm{Lm}(\omega) = 20 \lg |G(j\omega)| \qquad \text{[dB = decybel]} \tag{20}$$

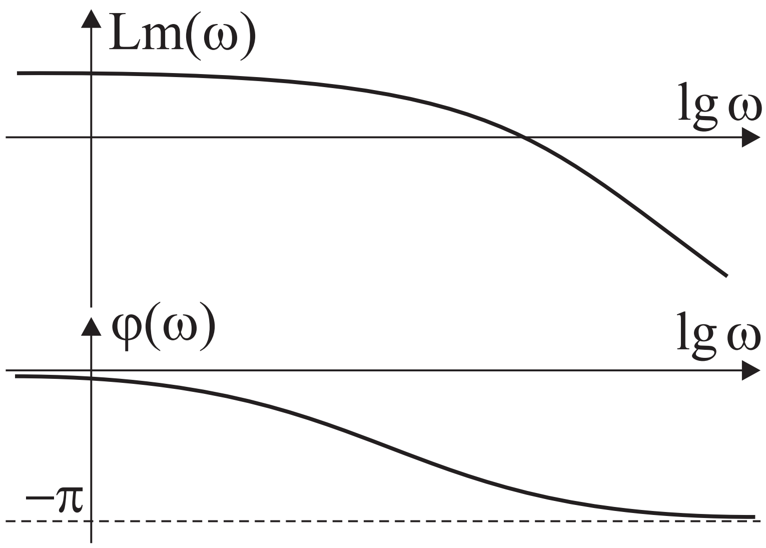
$$(\mathrm{Lm}(\omega) = 1[\mathrm{dB}] \to 20 \lg |G(j\omega)| = 1 \to$$

$$\to |G(j\omega)| = 10^{1/20} \approx 1{,}22 \to y_{\max}/u_{\max} \approx 1{,}22$$

$$\varphi(\omega) = \arg(G(j\omega)) \tag{21}$$



Rys. 28



Rys. 29

$$\text{oktawa} \to \omega_2 = 2\omega_1 \to \frac{\lg \omega_b - \lg \omega_a}{\lg 2} = 3{,}32 \lg \frac{\omega_b}{\omega_a} \tag{22}$$

$$\text{dekada} \to \omega_2 = 10\omega_1 \to \frac{\lg \omega_b - \lg \omega_a}{\lg 10} = \lg \frac{\omega_b}{\omega_a} \tag{23}$$

Zalety charakterystyk logarytmicznych

$$G(j\omega) = G_1(j\omega)G_(j\omega)G_3(j\omega)$$
$$\mathrm{Lm}(j\omega) = \mathrm{Lm}_1(j\omega) + \mathrm{Lm}_2(j\omega) + \mathrm{Lm}_3(j\omega)$$
$$\varphi(j\omega) = \varphi_1(j\omega) + \varphi_2(j\omega) + \varphi_3(j\omega)$$

Przykład (element inercyjny 1-go rzędu)

$$G(j\omega) = \frac{k}{1 + j\omega T}$$

$$|G(j\omega)| = \frac{|k|}{\sqrt{1+\omega^2 T^2}}, \qquad \varphi(\omega) = -\mathrm{arctg}(\omega T) \quad (k > 0)$$

$$\mathrm{Lm}(\omega) = 20\lg\frac{|k|}{\sqrt{1+\omega^2T^2}} = 20\lg|k| - 20\lg\sqrt{1+\omega^2T^2}$$

$$1+\omega^2T^2 = \begin{cases} 1 & \text{dla } \omega \ll 1/T \\ \omega^2T^2 & \text{dla } \omega \gg 1/T \end{cases}$$

$$1+\omega^2T^2 \approx \begin{cases} 1 & \text{dla } \omega < 1/T \\ \omega^2T^2 & \text{dla } \omega \geqslant 1/T \end{cases}$$

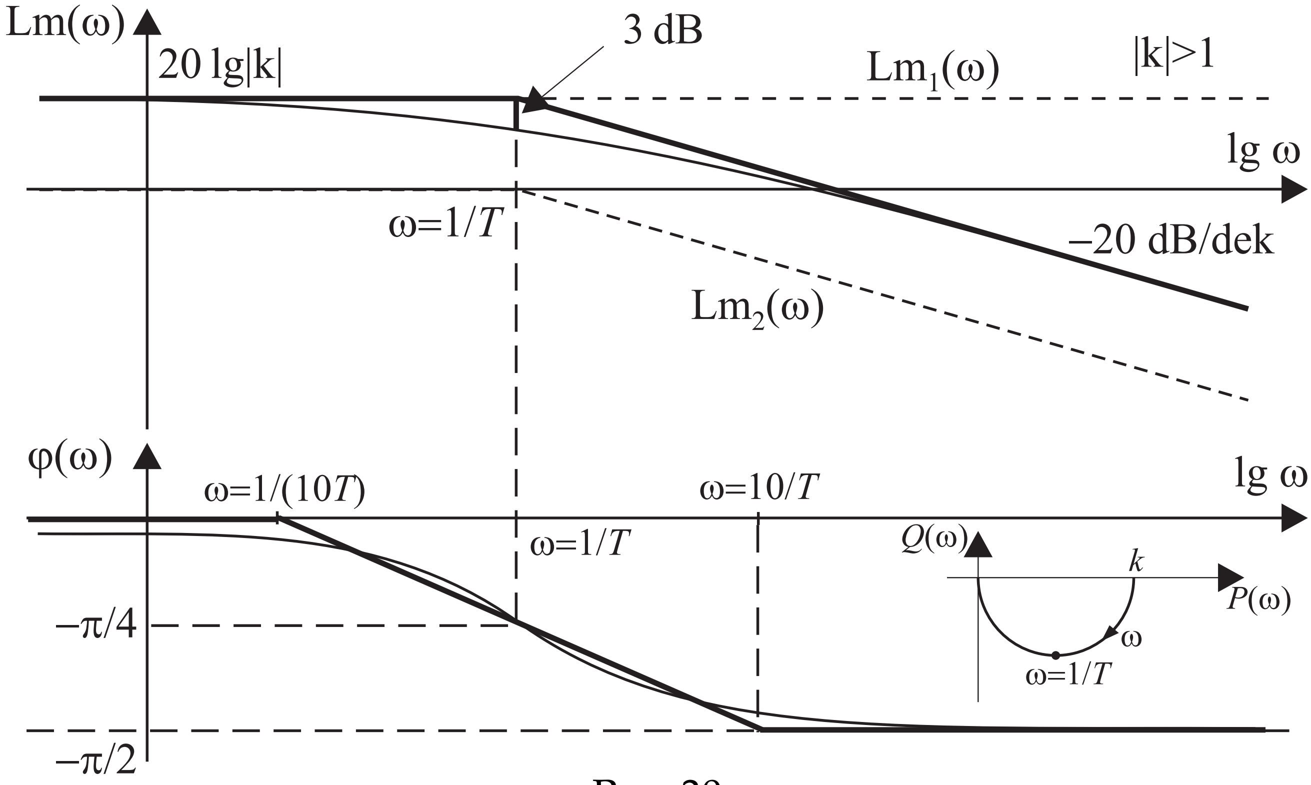
$$\mathrm{Lm}(\omega) \approx \begin{cases} 20\lg|k| - 20\lg 1 & \text{dla} \quad \omega < 1/T \\ 20\lg|k| - 20\lg\sqrt{\omega^2T^2} & \text{dla} \quad \omega \geqslant 1/T \end{cases} =$$

$$= \begin{cases} 20\lg|k| & \text{dla} \quad \omega < 1/T \\ 20\lg|k| - 20\lg(\omega T) & \text{dla} \quad \omega \geqslant 1/T \end{cases}$$

$$\mathrm{Lm}(\omega) = \mathrm{Lm}_1(\omega) + \mathrm{Lm}_2(\omega), \qquad \mathrm{Lm}_1(\omega) = 20\lg|k|$$

$$\mathrm{Lm}_2(\omega) = \begin{cases} 0 & \text{dla} \quad \omega < 1/T \\ -20\lg(\omega T) & \text{dla} \quad \omega \geqslant 1/T \end{cases}$$

$$-20\lg(\omega T) = -20\lg\omega + 20\lg(1/T) = 0 \quad \text{dla} \quad \omega = 1/T$$



Rys. 30

$$\mathrm{Lm}(10\omega_x) - \mathrm{Lm}(\omega_x) = 20\lg|k| - 20\lg(10\omega_x T) - 20\lg|k| +$$

$$+ 20\lg(\omega_x T) = 20\lg\frac{\omega_x T}{10\omega_x T} = -20\left[\frac{\text{dB}}{\text{dek}}\right] = -6\left[\frac{\text{dB}}{\text{okt}}\right]$$

$$\varphi(\omega) = -\mathrm{arctg}(\omega T) \quad \text{dla} \quad k > 0$$

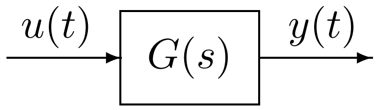
$$\varphi(1/T) = -\pi/4, \qquad \varphi(0) = 0, \qquad \varphi(\infty) \to -\pi/2$$

$$\Delta\mathrm{Lm}(\omega) = \mathrm{Lm}_{dokl}(\omega) - \mathrm{Lm}_{asympt}(\omega)$$

$$\Delta\mathrm{Lm}(1/T) = 20\lg|k| - 20\lg\sqrt{1 + (T/T)^2} -$$

$$- 20\lg|k| + 20\lg(T/T) = -20\lg\sqrt{2} = -3{,}03[\text{dB}]$$

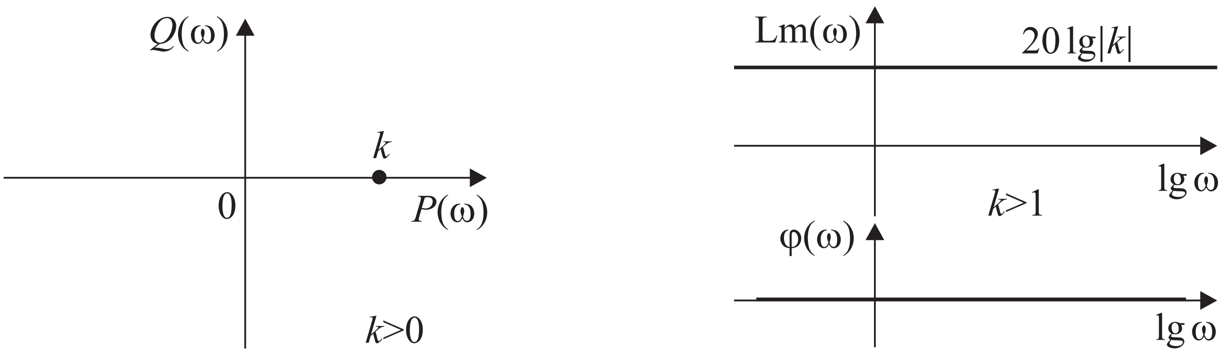
16. Charakterystyki czasowe i częstotliwościowe podstawowych układów dynamicznych



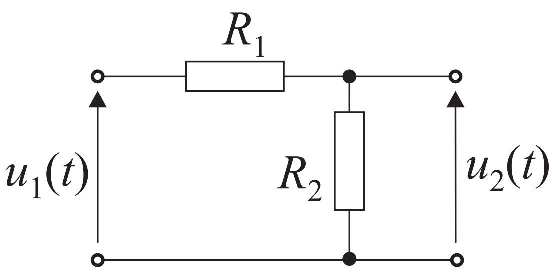
Rys. 31

(a) układ proporcjonalny $G(s) = \dfrac{Y(s)}{U(s)} = k$,

$$y(t) = ku(t), \qquad h(t) = \mathcal{L}^{-1}[G(s)/s] = k\mathbb{1}(t)$$



(a) (b)



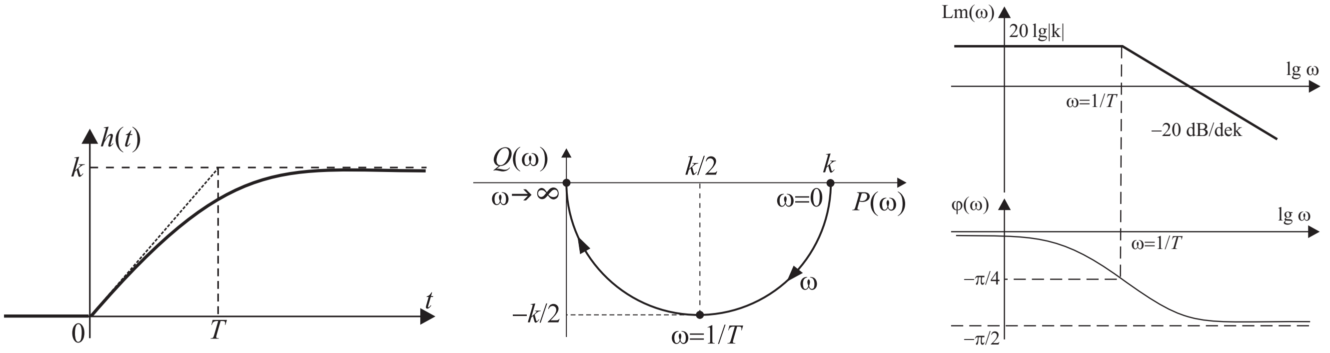
$$G(s) = \frac{U_2(s)}{U_1(s)} = \frac{R_2}{R_1 + R_2}$$

(c)

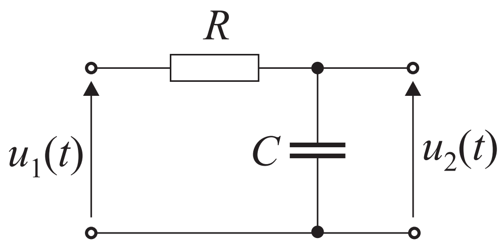
Rys. 32

(b) układ inercyjny 1-go rzędu $\quad G(s)=\dfrac{Y(s)}{U(s)}=\dfrac{k}{Ts+1}$

$$T\frac{dy(t)}{dt}+y(t)=ku(t), \quad h(t)=\mathcal{L}^{-1}[G(s)/s]=k\left(1-e^{-t/T}\right)\mathbb{1}(t)$$



(a) (b) (c)



$$G(s)=\frac{U_2(s)}{U_1(s)}=\frac{1}{1+sRC}$$

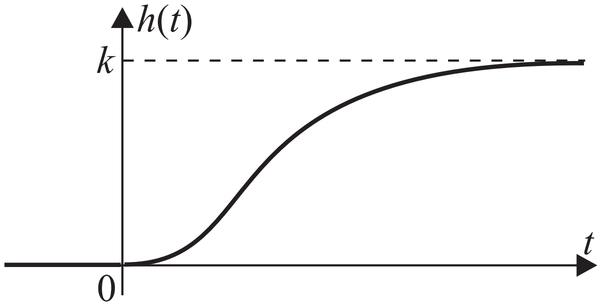
(d)

Rys. 33

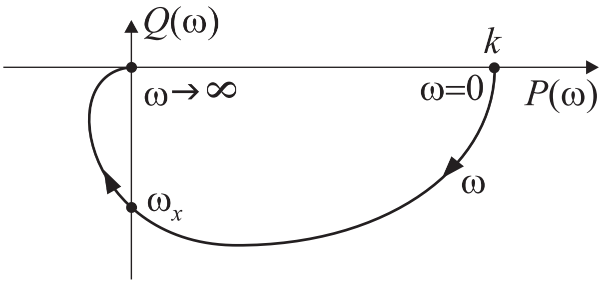
(c) ukł. inercyjny 2-go rzędu $\quad G(s)=\dfrac{Y(s)}{U(s)}=\dfrac{k}{(T_1s+1)(T_2s+1)}$

$$T_1T_2\frac{d^2y(t)}{dt^2}+(T_1+T_2)\frac{dy(t)}{dt}+y(t)=ku(t)$$

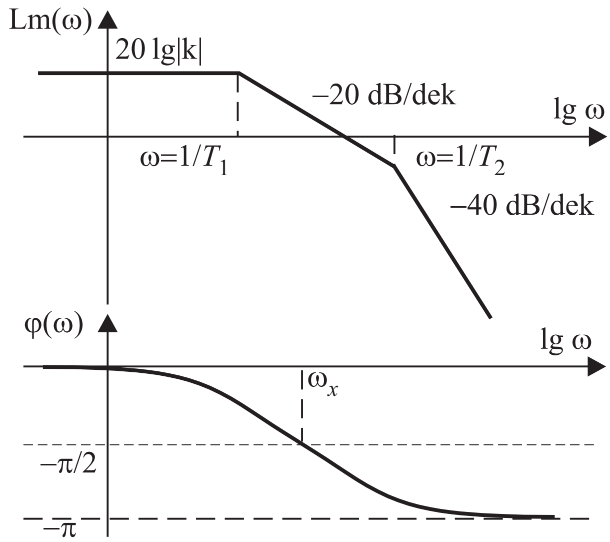
$$h(t)=\mathcal{L}^{-1}[G(s)/s]=k\left(1-\frac{T_1e^{-t/T_1}-T_2e^{-t/T_2}}{T_1-T_2}\right)\mathbb{1}(t)$$



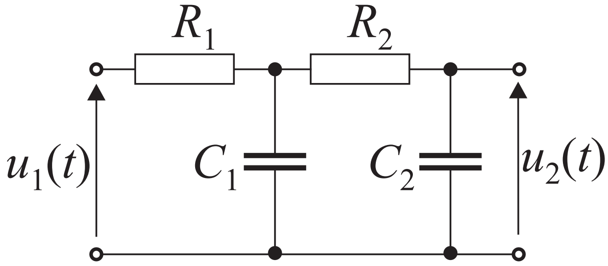
(a)



(b) $\omega_x = 1/\sqrt{T_1 T_2}$



(c) $T_1 > T_2$



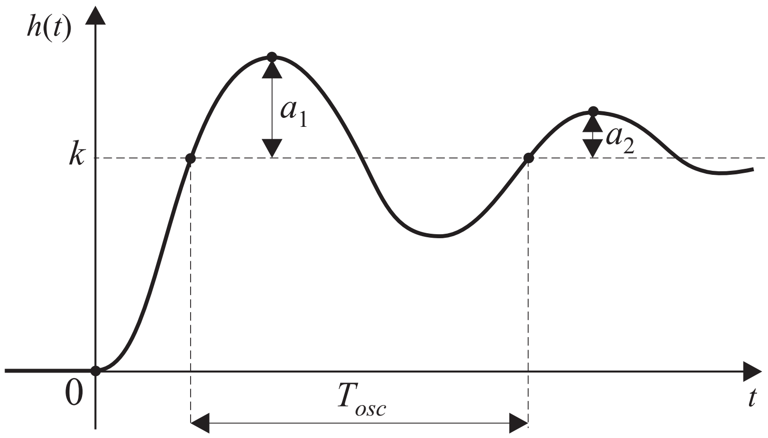
$$\text{(d)} \quad G(s) = \frac{U_2(s)}{U_1(s)} = \frac{1}{C_1R_1C_2R_2s^2 + (C_1R_1 + C_2R_2 + C_2R_1)s + 1}$$

Rys. 34

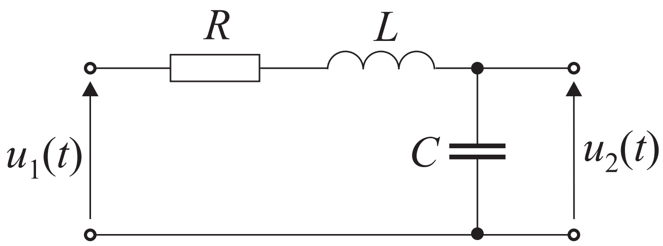
(d) układ oscylacyjny

$$G(s) = \frac{k}{T_n^2 s^2 + 2\zeta T_n s + 1} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{24}$$

k – wzmocnienie, T_n – stała czasowa,
$\omega_n = 1/T_n > 0$ – pulsacja drgań niet łumionych (p. naturalna),
ζ – współczynnik tłumienia (rozważamy $0 \leqslant \zeta < 1$),



(a)



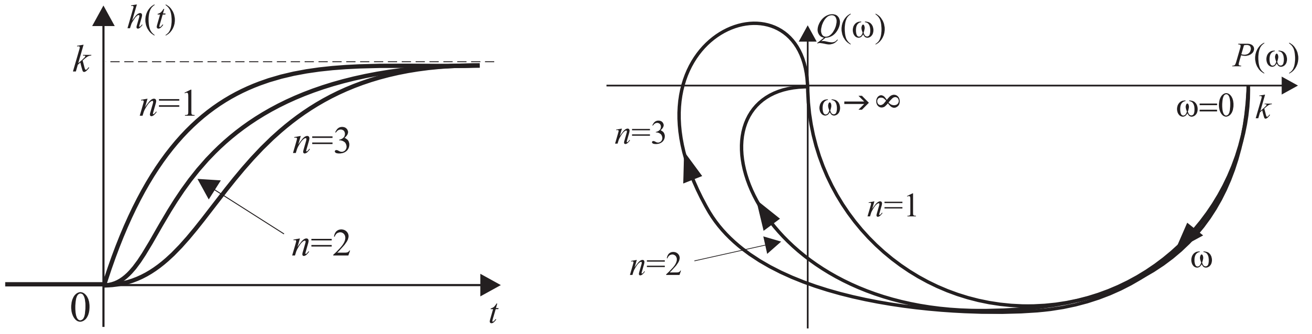
$$G(s) = \frac{U_2(s)}{U_1(s)} = \frac{1}{LCs^2 + RCs + 1}$$

(b)

Rys. 35

(e) układ inercyjny n-go rzędu

$$G(s) = \frac{k}{(1+sT_1)(1+sT_2)\dots(1+sT_n)} \quad \text{lub} \quad G(s) = \frac{k}{(1+sT)^n}$$



(a) (b)

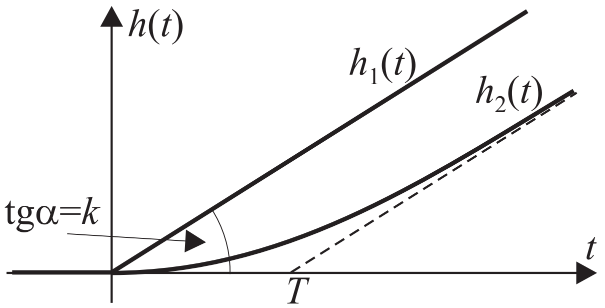
Rys. 36

(f) element całkujący idealny i rzeczywisty

$$G_1(s) = \frac{k}{s} \quad \rightarrow \quad H_1(s) = G_1(s)\frac{1}{s} = \frac{k}{s^2} \quad \rightarrow \quad h_1(t) = kt\mathbb{1}(t)$$

$$G_2(s) = \frac{k}{s(1+sT)} \quad \rightarrow \quad H_2(s) = G_2(s)\frac{1}{s} = \frac{k}{s^2(1+sT)} \quad \rightarrow$$

$$\rightarrow \quad h_1(t) = k\left(t - T + Te^{-t/T}\right)\mathbb{1}(t) = k\left[t - T\left(1 - e^{-t/T}\right)\right]\mathbb{1}(t)$$

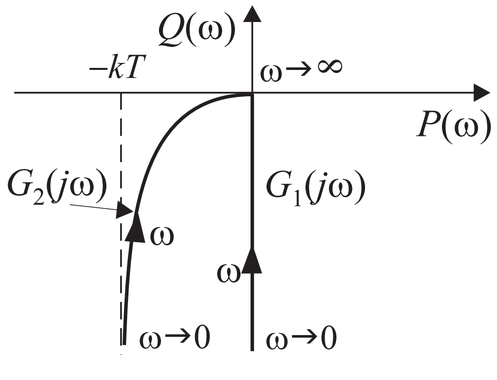


Rys. 37

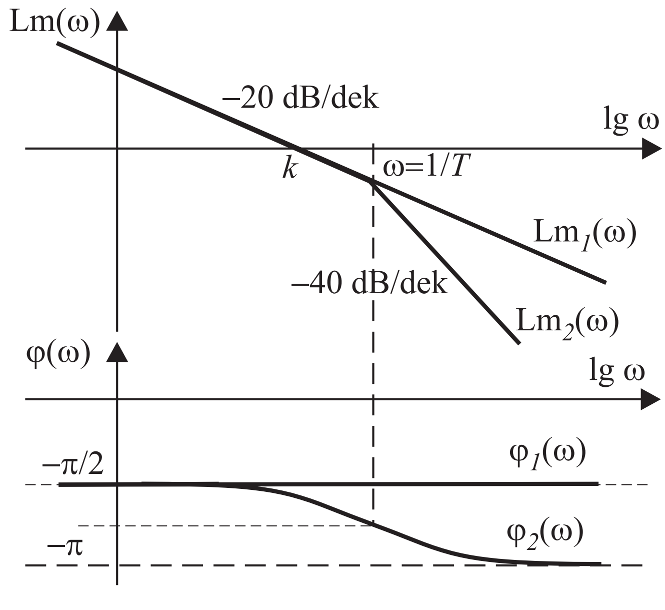
charakterystyki amplitudowo-fazowe i logarytmiczne Bodego:

$$G_1(j\omega) = \frac{k}{j\omega} = -j\frac{k}{\omega} \quad \rightarrow \quad G_1(j0) \rightarrow 0{-}j\infty, \;\; G_1(j\infty) \rightarrow 0{+}j0$$

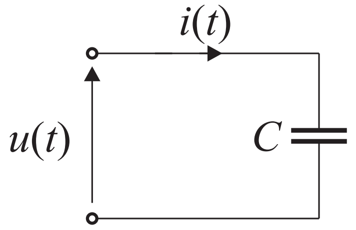
$$G_2(j\omega) = \frac{k}{j\omega(1+j\omega T)} = \frac{-k(\omega^2 T + j\omega)}{\omega^2(1+\omega^2 T^2)} = $$
$$= \frac{-kT}{1+\omega^2 T^2} + j\frac{-k}{\omega(1+\omega^2 T^2)}$$
$$G_2(j0) \rightarrow -kT - j\infty, \qquad G_2(j\infty) \rightarrow 0 + j0$$



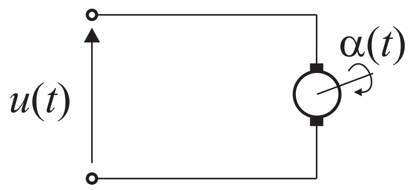
(a)



(b)



(c) $G(s) = \frac{U(s)}{I(s)} = \frac{1}{sC}$



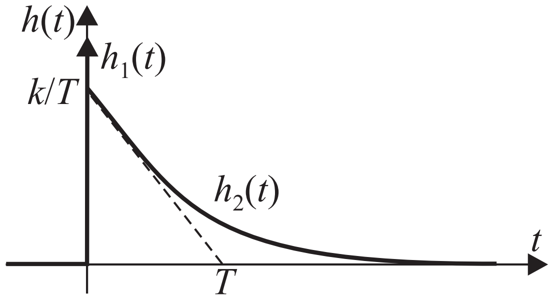
(d) $G(s) = \frac{A(s)}{U(s)} = \frac{k}{s(1+sT)}$

Rys. 38

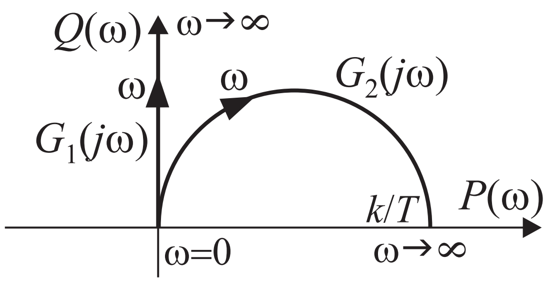
(g) element różniczkujący idealny i rzeczywisty

$$G_1(s) = ks \quad \rightarrow \quad H_1(s) = ks\frac{1}{s} = k \quad \rightarrow \quad h_1(t) = k\delta(t)$$

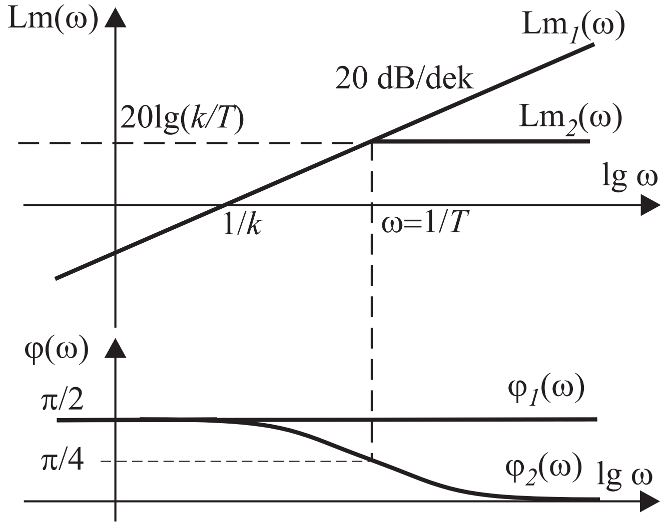
$$G_2(s) = \frac{ks}{1+sT} \quad \rightarrow \quad H_2(s) = \frac{ks}{1+sT}\frac{1}{s} \quad \rightarrow \quad h_2(t) = \frac{k}{T}e^{-t/T}\mathbb{1}(t)$$



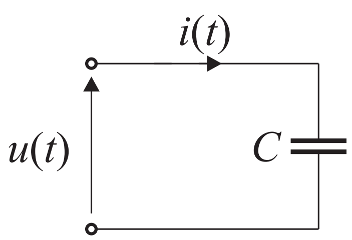
(a)



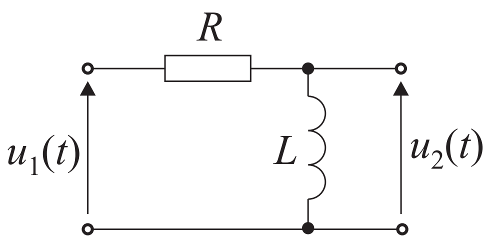
(b)



(c)



(d) $G(s) = \dfrac{I(s)}{U(s)} = sC$



(e) $G(s) = \dfrac{U_2(s)}{U_1(s)} = \dfrac{sL}{sL+R}$

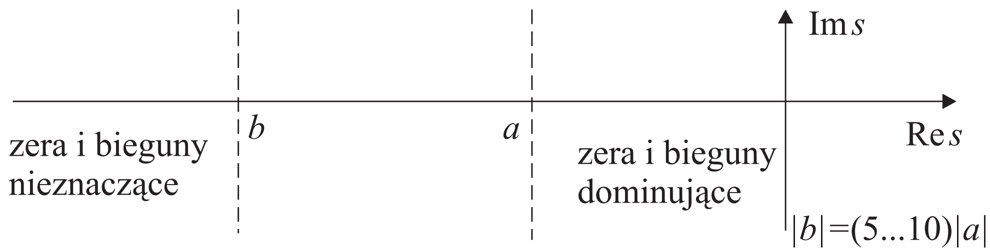
Rys. 39

17. Redukcja rzędu modelu układu dynamicznego

$$G(s) = \frac{k}{(1+sT_1)(1+sT_2)} \to h(t) = k\left(1 - \frac{T_1 e^{-t/T_1} - T_2 e^{-t/T_2}}{T_1 - T_2}\right)\mathbb{1}(t)$$

$s_1 = -1/T_1, s_2 = -1/T_2,$ zał. $T_2 \ll T_1 \to s_2 \ll s_1$

$$h(0) = 0, h(\infty) = k \to h_{red}(t) = k(1 - e^{-t/T_1})\mathbb{1}(t) \to G_{red} = \frac{k}{1+sT_1}$$

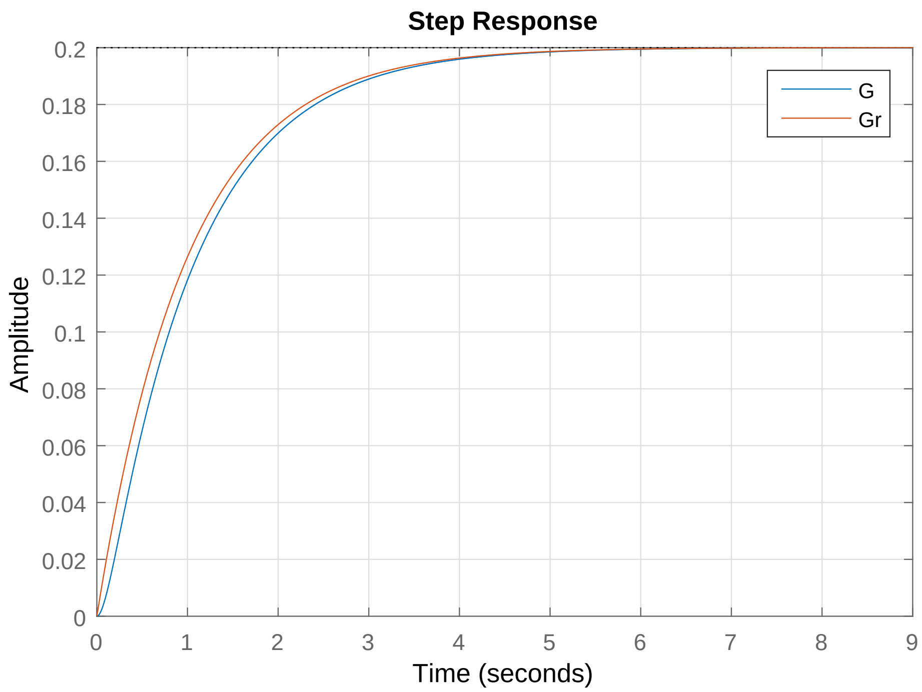


Rys. 40

Przykład (element inercyjny 2-go rzędu)

$$G(s) = \frac{2}{(s+1)(s+10)} = \frac{2}{10(s+1)(0,1s+1)}$$

$$G_r(s) = \frac{2}{10(s+1)}$$



Rys. 41