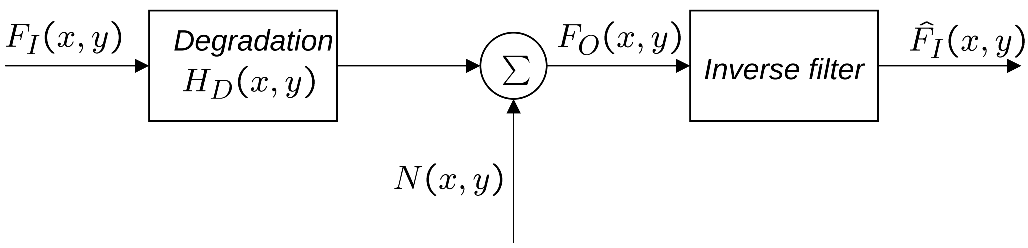
Image filtering

- Classification of the image filtering methods
  - Filtering in spatial and frequency domain
  - Linear and nonlinear filtering methods
- The main tasks of image filtration
  - Removing of unwanted noise in the image
  - Improving the sharpness
  - Extraction of some features
  - Removal of certain defects in the image
  - Improving the image of poor technical quality
  - Image reconstruction in the case of its degradation
- Linear and nonlinear filters in spatial domain - image filtering is done using neighborhood pixels values processing
  - Linear filters are based on linear operations (easy implementation and understanding). Filter is linear if processing function meets conditions:
    - is additive $\phi(A+B) = \phi(A) + \phi(B)$
    - is homogeneous $\phi(\alpha A) = \alpha\phi(A), \quad \alpha > 0$
  - Nonlinear filter sometimes offer better performance, but they are not always predictable and more difficult in implementation

The concept of image filtering



- Restoration of degraded image is done by applying the appropriate inverse filter according to the degradation function
- As the output estimation of reconstructed image is given
- The basic problem - the lack of knowledge of the degradation and distortion functions (one can not build real inverse filter which exactly corresponds to degradation functions)
- In practice, the methods of image reconstruction and quality improvement rely on heuristic approximation of unknown inverse filter

Image filtration – linear filters

- Convolution
  - For two dimensional image function $J(x,y)$ the spatial filtering is done by convolving a filter mask (convolution kernel) with an image

$$J_w(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} J(x-i, y-j)w(i,j)$$

$w(i,j)$ - window of the filter (kernel) with filter weights and dimension *2a+1* by *2b+1*

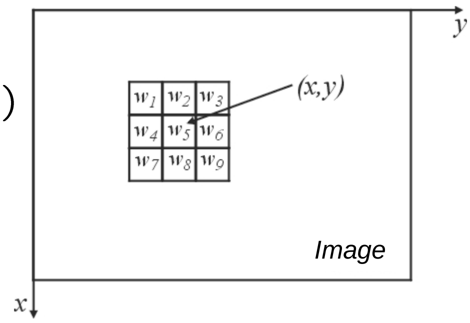
  - Properties of the convolution (useful in practical implementation):
    - **associative** – it allows for separation of arbitrarily large filtering mask for subsequent filtering using small masks
    - **separation** – it allows to replace the two-dimensional image filtering by combination of one dimensional filtering

Image filtration – linear filters

- Convolution for 3x3 filter mask (kernel)

$$J_w(x,y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} J(x-i, y-j)w(i,j)$$

$$a = 1,\ b = 1$$

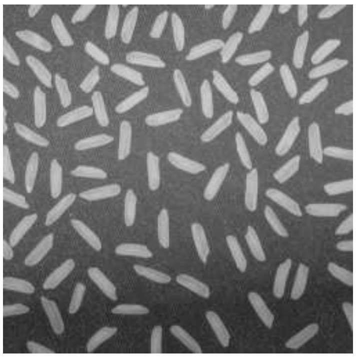


$$J_w(x,y) = w_1 J(_{x+1,y+1}) + w_2 J(_{x+1,y}) + w_3 J(_{x+1,y-1}) +$$
$$+ w_4 J(_{x,y+1}) + w_5 J(_{x,y}) + w_6 J(_{x,y-1}) +$$
$$+ w_7 J(_{x-1,y+1}) + w_8 J(_{x-1,y}) + w_9 J(_{x-1,y-1})$$

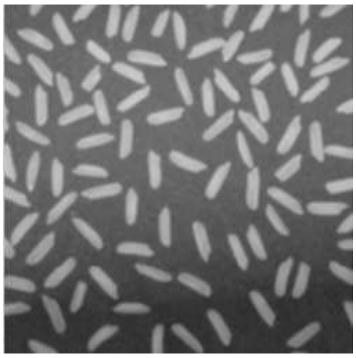
- The response of linear spatial filtering is given by a sum of product of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask.
- Practical implementation of convolution requires the a new buffer for the output image (contrary to the point operations)!

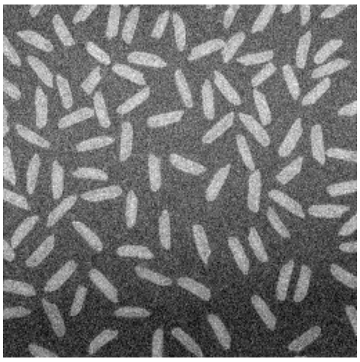
Image filtration – linear filters

– Example of lowpass filtering



Averaging filter 3x3





Gaussian filter

3x3

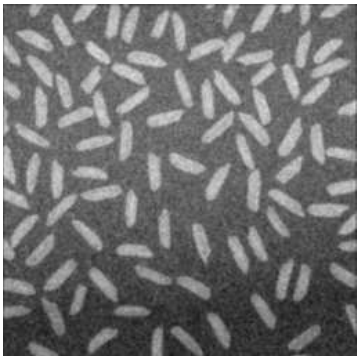
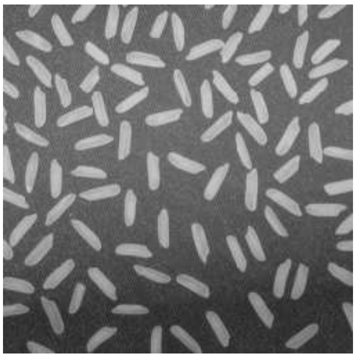
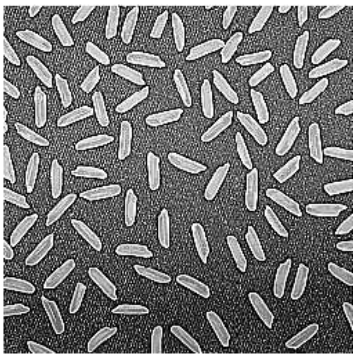


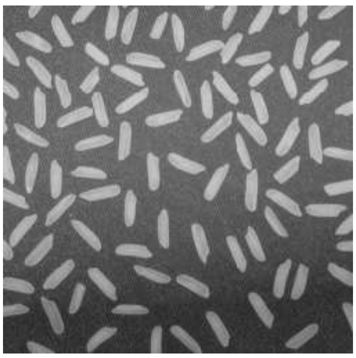
Image filtration – linear filters

– Example of highpass filtering



Sharpening filter





Sobel gradients operators

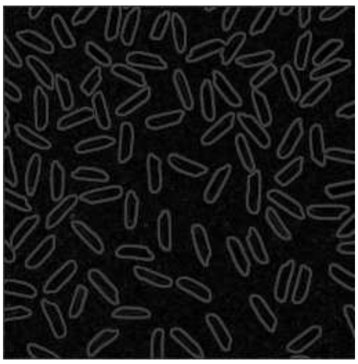


Image filtration – nonlinear filtering

- Logical filters
  - Logical filtering is the simplest example of a nonlinear filtering. It is based on the checking of the logical expression that describes the relationship between the pixels of an arbitrary neighborhood depending on your needs. Most often four or eight neighborhood is analyzed and it is used primarily for binary images.

	a	
b	*X*	*c*
	d	

  - Examples of rules
    - Eliminating noises in the form of isolated points and horizontal lines one pixel wide

$$X' = \begin{cases} a, & if\ a = d \\ X, & in\ other\ case \end{cases}$$

    - Eliminating of isolated single points

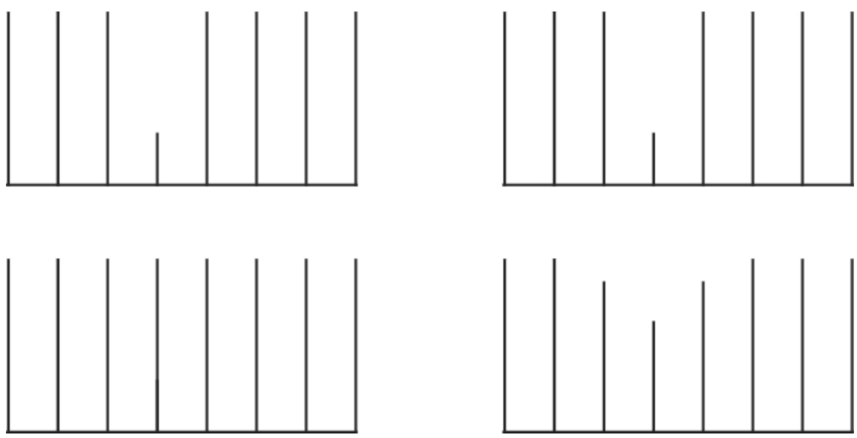
$$X' = \begin{cases} a, & if\ a = b = c = d \\ X, & in\ other\ case \end{cases}$$

Image filtration – nonlinear filtering

- Order statistic filters – Median filtering
  - The resulting value of the point is the median (middle value) of the set of points in the neighborhood considered for filtration
  - The advantage of the median filter - the ability to remove most of random noise like "impulse noise" or "salt and pepper noise". Median filters cause considerably less blurring of edges and details in comparison to linear smoothing filters and convolution methods
  - Median filtering for noise reduction

Before filtration

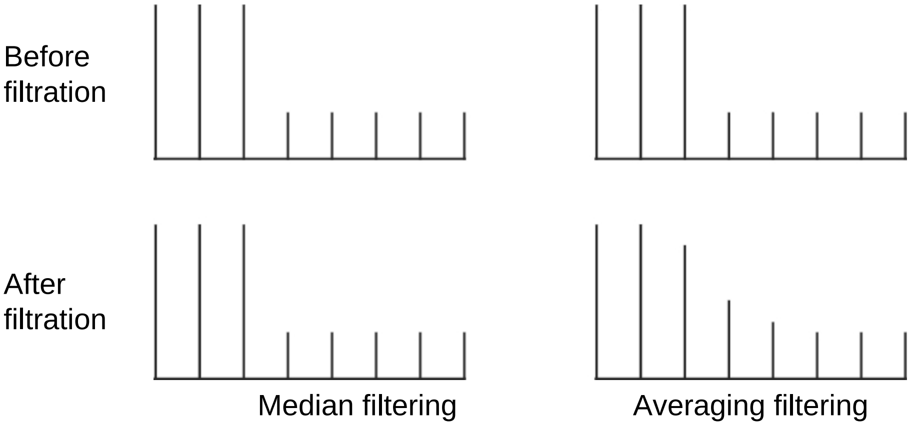
After filtration



Median filter   Averaging filter

Image filtration – nonlinear filtering

– Median filtering of edges in the image



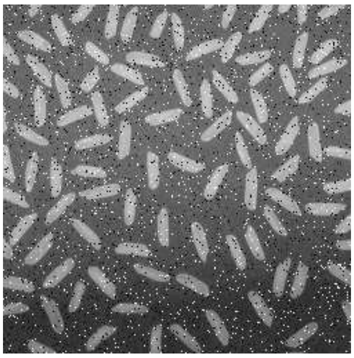
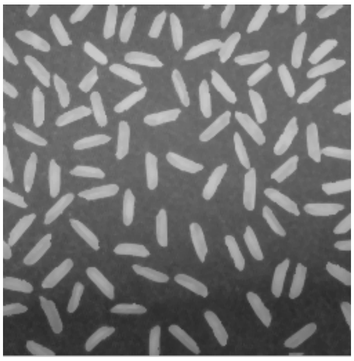
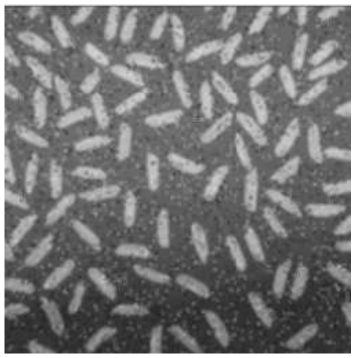
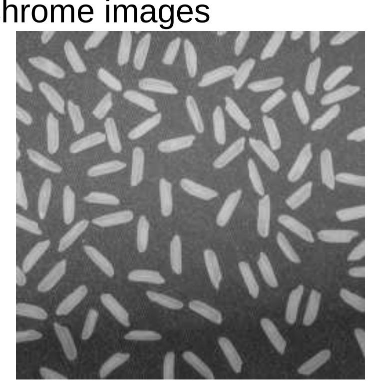




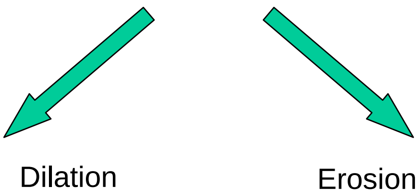
Image filtration – nonlinear filtering

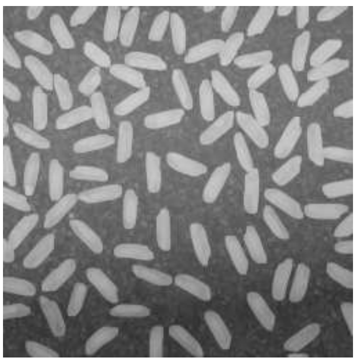
- Order statistic filters – local maximum and minimum filters
  – They are used as the basic morphological operators respectively dilation and erosion on monochrome images



Structuring element

1	1	1
1	**1**	1
1	1	1



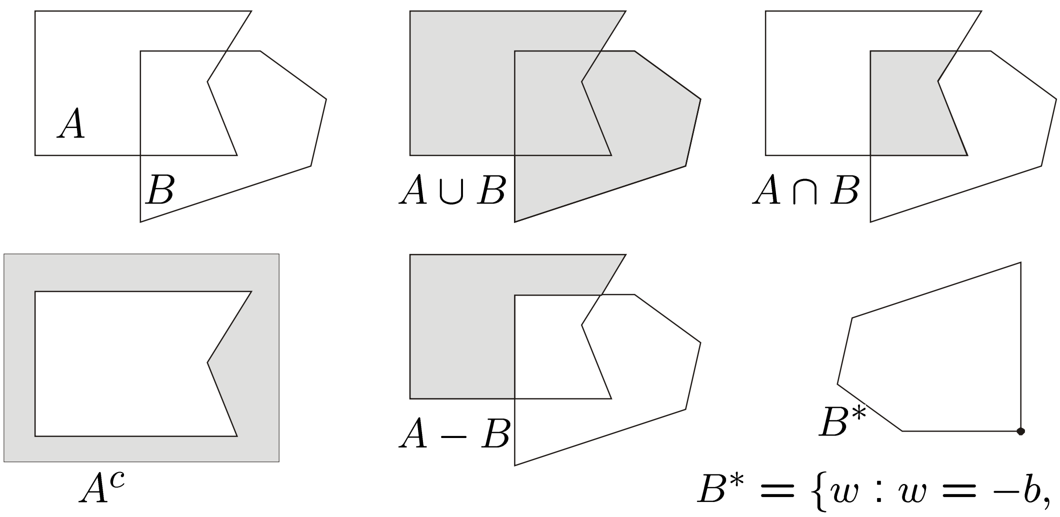



Morphological image processing

- Morphological image transformations are derived from mathematical morphology – the section of mathematics based on set theory
- They are used for morphological filtering, information finding and shape analysis using structuring elements
- Morphological operations was used originally in the processing of binary images, extension of the definition of two basic operations enables the processing of gray scale images
- Basic operations can be classified as nonlinear context processing
- Two basic morphological operations: dilation and erosion

Morphological image processing

- Some basic concepts from set theory



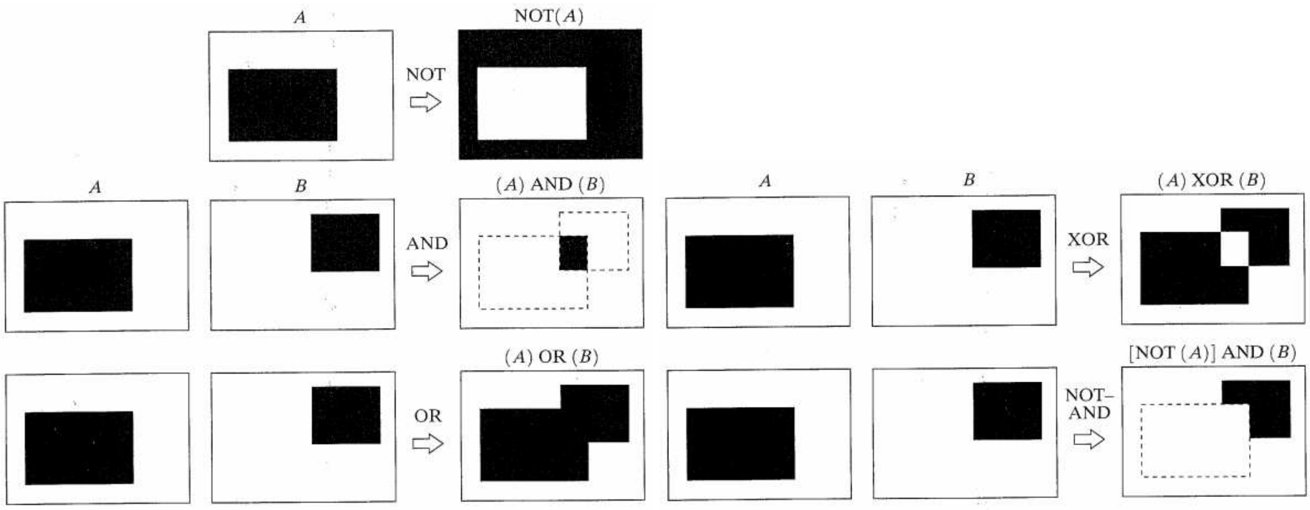
$$B^* = \{w : w = -b, b \in B\}$$

$$A^c = \{w : w \notin A\}$$

$$A - B = \{w : w \in A, w \notin B\} = A \cap B^c$$

Morphological image processing

- Logic operation on binary images



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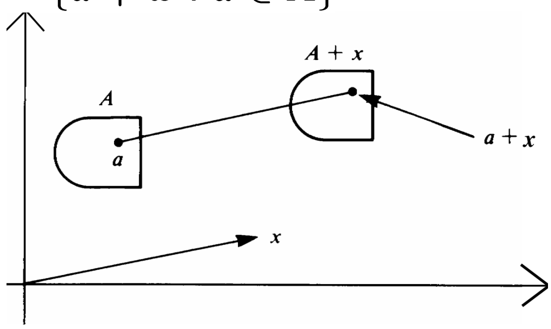
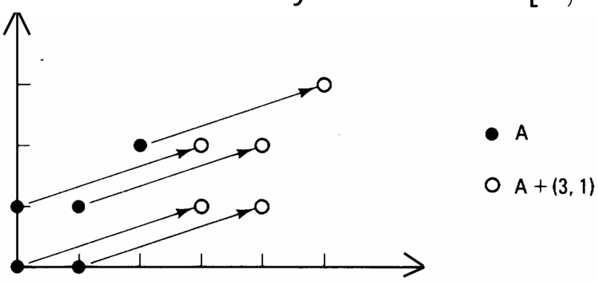
Morphological image processing

- Translation of the set
  - Translation of set $A \subset \mathbb{R}^2$ by vector $\boldsymbol{x} \subset \mathbb{R}^2$ is defined as

$$A + \boldsymbol{x} = \{a + \boldsymbol{x} : a \in A\}$$



  - Translation of discrete set by vector $\boldsymbol{x} = [3, 1]$

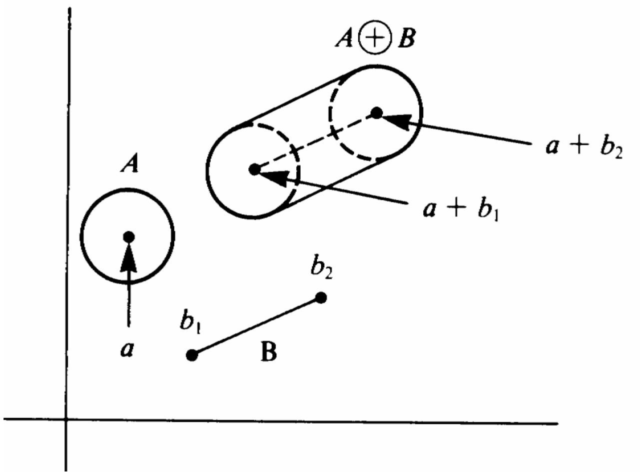


Morphological image processing

- Dilation of the set
  - Dilation of set A by B $(A, B \subset \mathbb{R}^2)$, where B is structuring element can be defined as

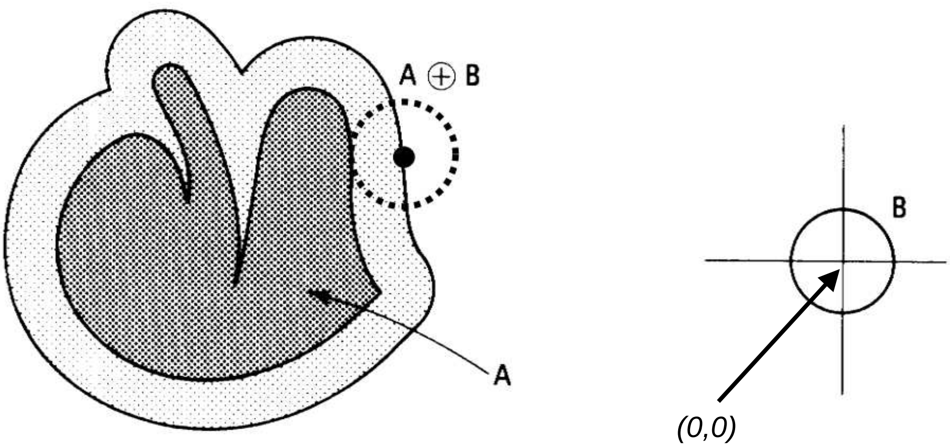
$$A \oplus B = \bigcup_{b \in B} (A + b)$$

  - Dilation is the set of all displacements (union) of set A by elements of set B



Morphological image processing

- Example of set dilation

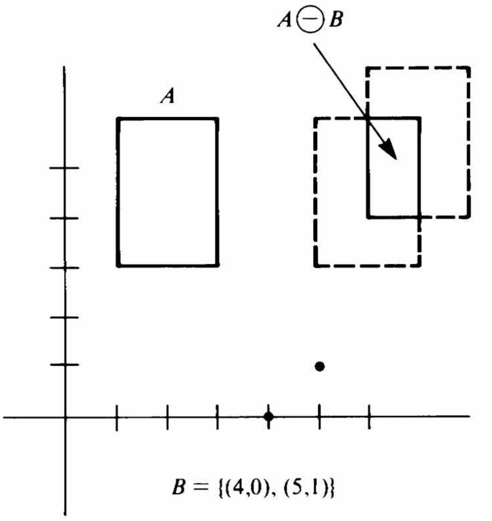


Morphological image processing

- Erosion of the set
  - Erosion of set A by B $(A, B \subset \mathbb{R}^2)$, where B is structuring element can be defined as

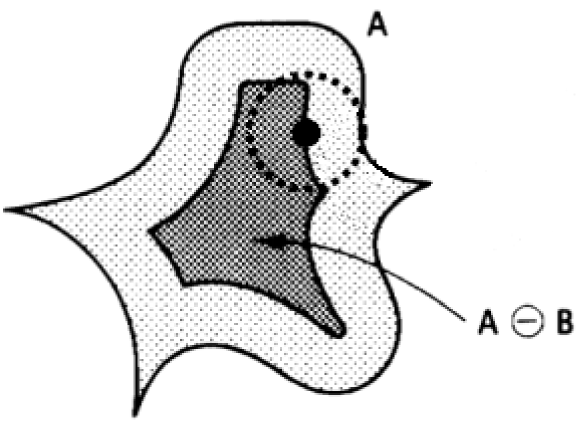
$$A \ominus B = \bigcap_{b \in B} (A + b)$$

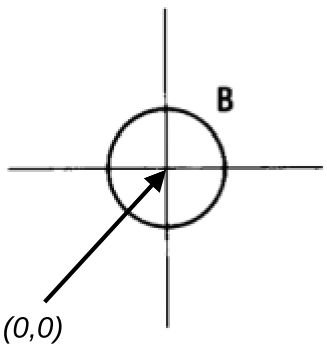
  - Erosion is intersection set of all translations of set A by elements of set B



Morphological image processing

- Example of set erosion





Morphological image processing

- Morphological operations for binary images
  - Structuring element

1	1	1
0	**1**	0
1	1	1

B

  - The selected point (origin) of the structuring element B is moved on successive image pixels
  - In each pixel (x, y) of the image specified logical operations are performed using neighbor points B_{xy} defined by structuring element B
  - The size and shape of points in structuring element determine the range and nature of the processing of morphological operations
  - There is a problem with the boundary pixels of the image (as in the case of the typical linear image filtering)

Morphological image processing

- Dilation

$$A \oplus B = \{x, y : B_{xy} \cap A \neq \emptyset\}$$

- Example

0	1	1
0	**1**	0
0	1	1

Structuring element

0	0	1	0	0
0	1	1	1	0
0	1	1	0	0
0	0	1	1	0
0	0	0	0	0

Source image

0	1	1	1	1
0	1	1	1	0
0	1	1	1	1
0	1	1	1	0
0	0	1	1	1

Resulting image

Morphological image processing

– Erosion

$$A \ominus B = \{x, y : B_{xy} \subseteq A\}$$

– Example

0	1	1
0	**1**	0
0	1	1

Structuring element

0	0	1	0	0
0	1	1	1	0
0	1	1	0	0
0	0	1	1	0
0	0	0	0	0

Source image

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Resulting image

Morphological image processing

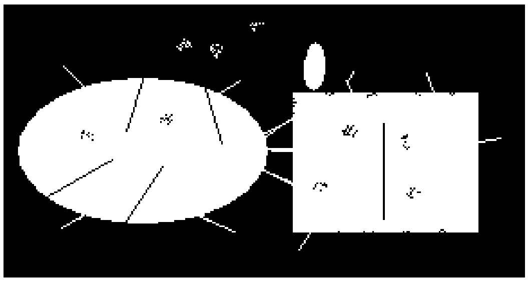
- Properties of dilation
  – Commutative operation $A \oplus B = B \oplus A$
  – Invariant because of the translation $A_T \oplus B = (A \oplus B)_T$
  – Extending the object (extensive) $A \subseteq Y \Rightarrow A \oplus B \subseteq Y \oplus B$
  – Successive dilation of the object A by B , and then by C is equivalent to the dilation of A by $B \oplus C$ (associative property)
  $$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

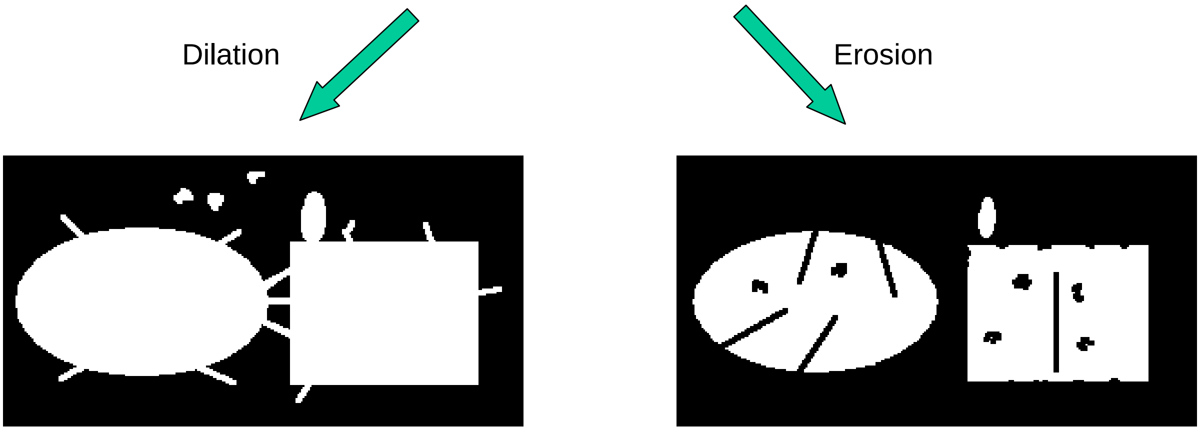
- Properties of erosion
  – Dual operation to dilation with respect to set complementation and reflection (in general erosion and dilation operations are not reversible)
  $$(A \ominus B)^c = A^c \oplus B^*$$
  – Erosion is not commutative $A \ominus B \neq B \ominus A$
  – Invariant because of the translation
  – Is not extensive
  – Associative property $(A \ominus B) \ominus C = A \ominus (B \oplus C)$

Morphological image processing

1	1	1
1	**1**	1
1	1	1

Structuring element





Morphological image processing

- Opening and closing operations (two fundamental morphological filters)
  - Opening

    $$(A \circ B) = (A \ominus B) \oplus B$$

    - Opening is not extensive

      $$A \circ B \subseteq A$$

    - Iterative repeating of the opening does not change the result

      $$A \circ B = (A \circ B) \circ B$$

  - Closing

    $$(A \bullet B) = (A \oplus B) \ominus B$$

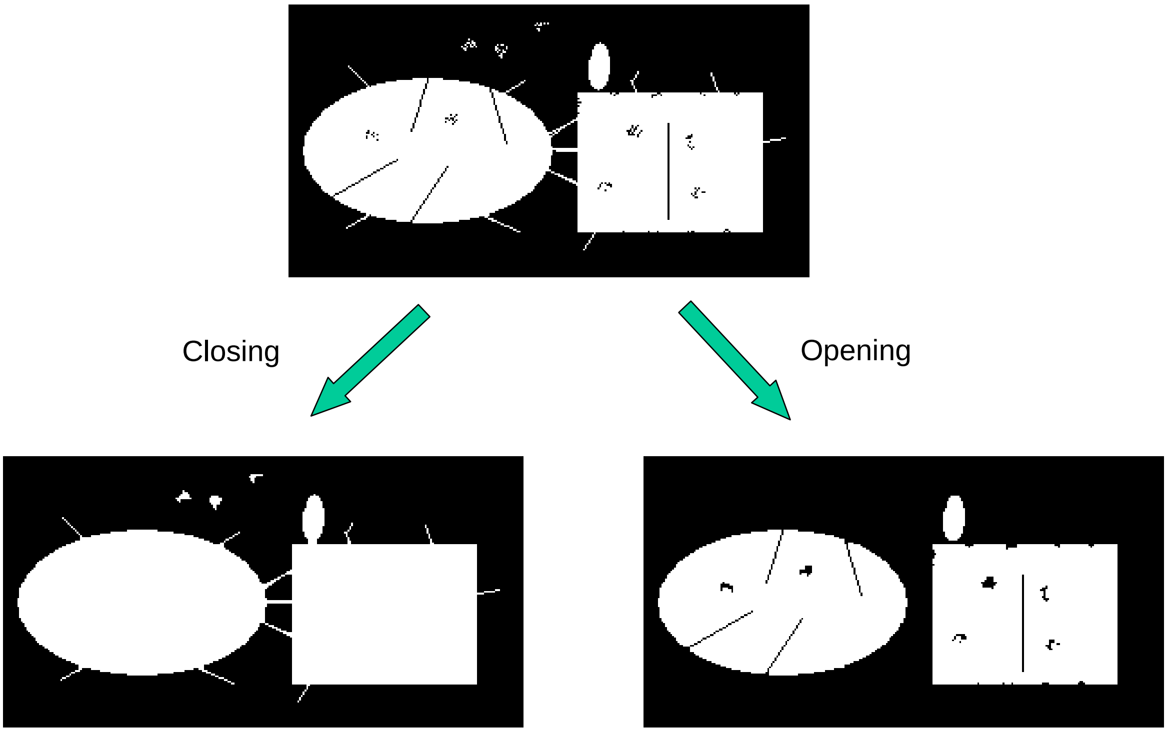
    - Closing is extensive

      $$A \subseteq A \bullet B$$

    - Iterative repeating of the closing does not change the result (the same as opening)

      $$A \bullet B = (A \bullet B) \bullet B$$

Morphological image processing



Morphological image processing

- The Hit-or-Miss transformation
  - It can be used as a basic tool for shape detection of selected pattern

$$A \otimes B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

$$X-\text{pattern},\ W-\text{analyzed window}$$

$$B_1 = X,\ B_2 = W - X \ \Rightarrow \ B = (B_1, B_2)$$

$$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

  - Hit-or-Miss with using erosion and dilation

$$A \otimes B = (A \ominus B_1) - (A \oplus B_2^*)$$

Morphological image processing

– Example of Hit-or-Miss transformation

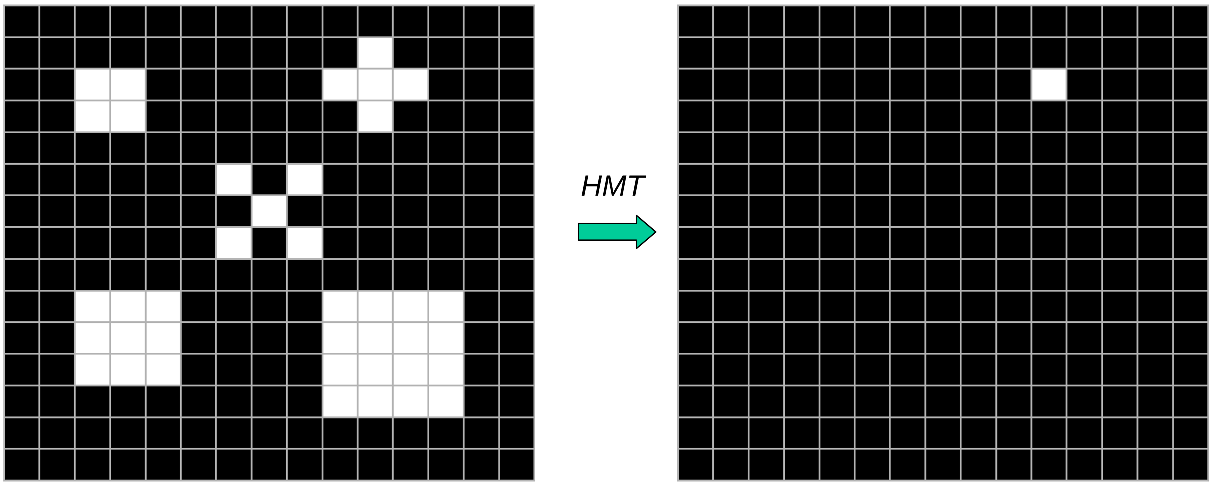
B_1

0	0	0	0	0
0	0	1	0	0
0	1	**1**	1	0
0	0	1	0	0
0	0	0	0	0

B_2

1	1	1	1	1
1	1	0	1	1
1	0	**0**	0	1
1	1	0	1	1
1	1	1	1	1

Matlab ->



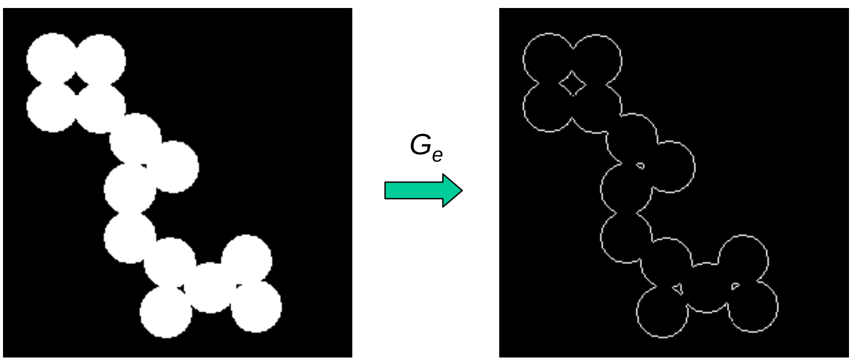
Morphological image processing

- Boundary extraction
  – With using erosion
  $$G_e(A) = A - (A \ominus B)$$
  – With using dilation
  $$G_d(A) = (A \oplus B) - A$$
  – Morphological gradient
  $$M_{grad}(A) = (A \oplus B) - (A \ominus B)$$

B - structuring element representing a discrete form of the unit circle, in practice it is approximated by a square 3x3

0	1	0
1	**1**	1
0	1	0

B



Morphological image processing

- Geodesic dilation and extraction of connected components (reconstruction by dilation)
  - Geodesic dilation

$$D_g(J) = (J \oplus B) \cap A$$

J - marker (pixel or pixels belonging to the objects)

A - mask (typically source image)

B - structuring element

  - Reconstruction using geodesic dilation
    - Reconstruction $R(J)$ of the mask A starting from marker J is obtained by iteratively performing geodesic dilation until the result becomes stable

$$R(J) = D_g^{(i)}(J)$$

i - the smallest value such that $D_g^{(i)}(J) = D_g^{(i+1)}(J)$

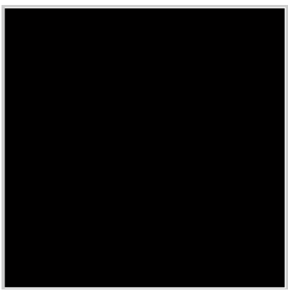
Morphological image processing

- An example of the use of reconstruction for the detection and removal of objects intersecting borders of the image

$$O_w = A - R(J) = A - D_g^{(i)}(J)$$

A

J



B

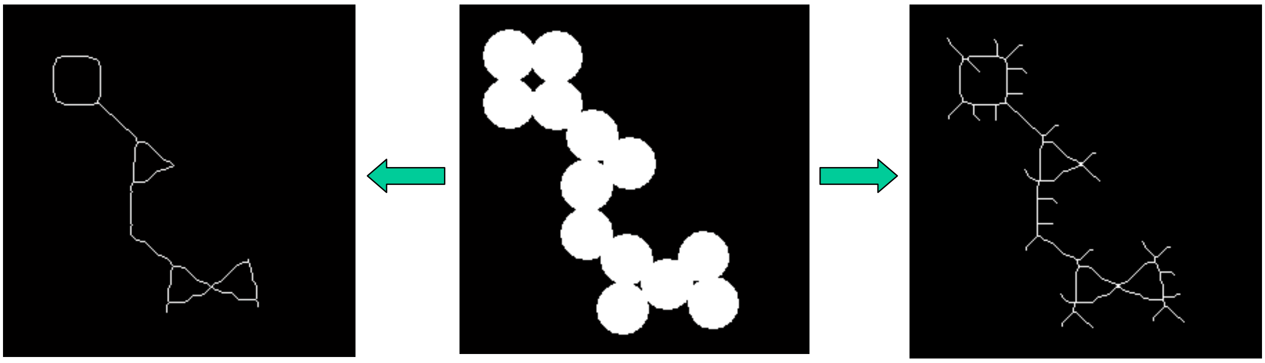
0	1	0
1	**1**	1
0	1	0

$R(J)$

O_w

Morphological image processing

- Other morphological transformations
  - Thinning
  - Thickening (dual to thinning)
  - Skeletonizing
  - Other operations related to the removal of objects pixels

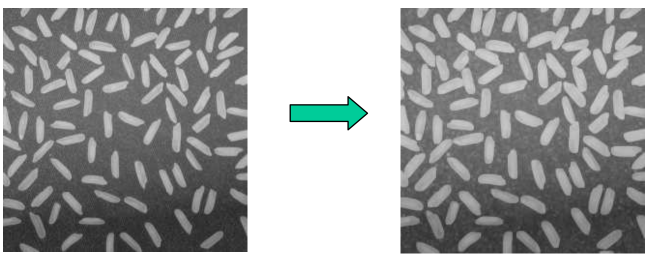


Thinning *Skeleton*

Morphological image processing

- Morphological operations on grayscale images
  - Basic operations of erosion and dilation
    - Dilation

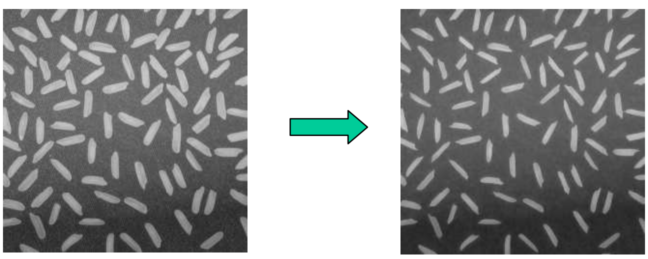
$$L'(m,n) = L \oplus B = max\{L(i,j) \text{ dla } i,j \in B(m,n)\}$$



1	1	1
1	**1**	1
1	1	1

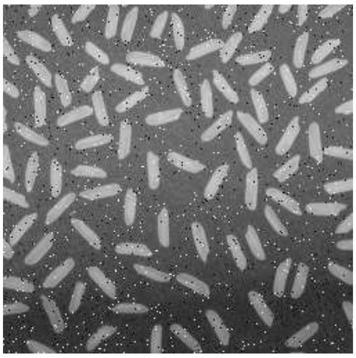
- Erosion

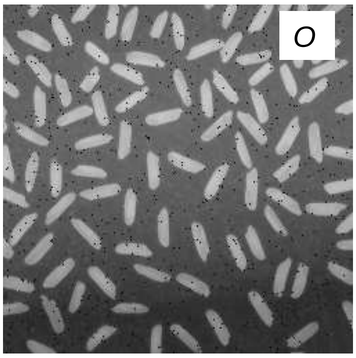
$$L'(m,n) = L \ominus B = min\{L(i,j) \text{ dla } i,j \in B(m,n)\}$$

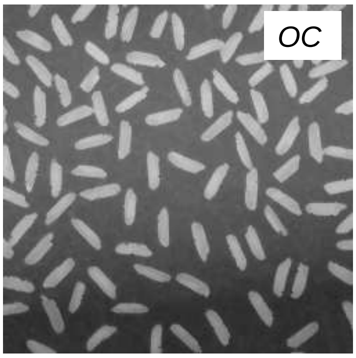


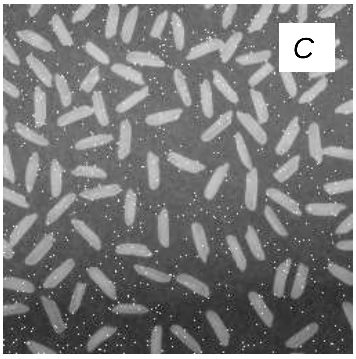
Morphological image processing

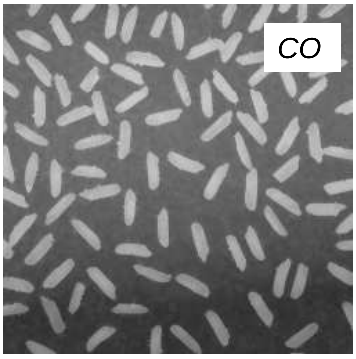
– Opening O and closing C, OC and CO filters







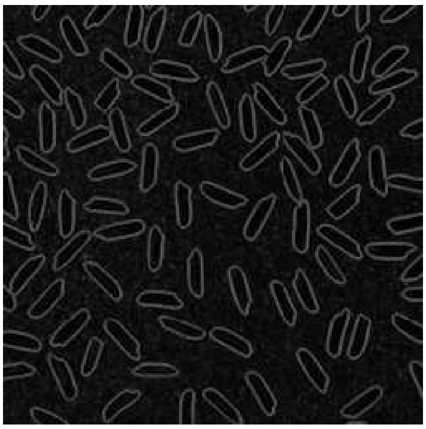




Morphological image processing

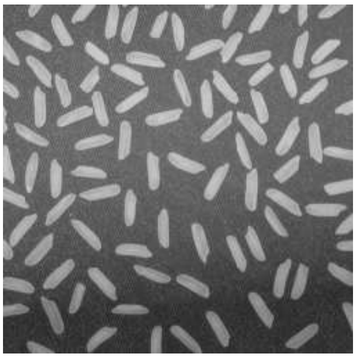
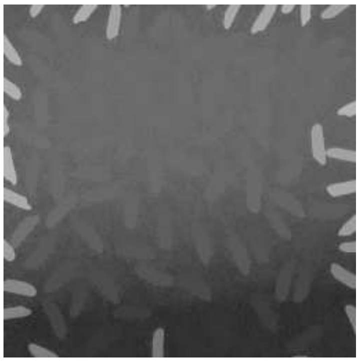
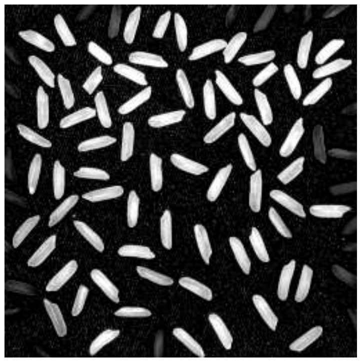
– Detection of contours - morphological gradient

$$M_{grad}(A) = (A \oplus B) - (A \ominus B)$$



Morphological image processing

– Morphological reconstruction

```
I = imread('rice.png');
mark=I-20;
mark(2:255,2:255)=0;
r=imreconstruct(mark,I,4);
imshow(r)
In=I-r;
figure, imshow(imadjust(In))
```

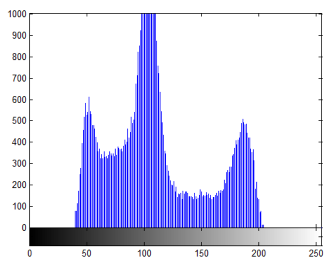
imclearborder

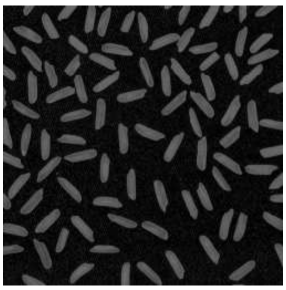
Morphological image processing

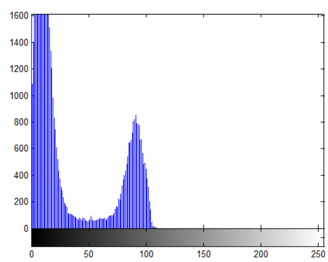
- Top-Hat and Bottom-Hat transformations
  - Top-Hat – unification of dark background with emphasis objects that are smaller then structuring element and brighter then their surroundings

$$TH = A - (A \circ B)$$

B - circular mask with a radius of 12







  - Bottom-Hat – emphasis of objects that are darker then their surroundings

$$BH = (A \bullet B) - A$$

Morphological image processing

– Top-Hat and Bottom-Hat transformations can be used to improve the contrast of the image

$J = A + TH - BH$ B - structuring element 9x9